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ABSTRACT

This study examined the role and development of applied problem solving in mathematics education in the Finnish comprehensive school. The development during the 1980s of applied problem solving in schools was positive in many ways: the changes were cautious and took place more slowly than had been planned; and many obstacles related to the organization of instruction, the curriculum, and the teaching culture constrained a faster and more far-reaching development. In the early 1980s, applied problem solving was intended to be realized mainly by increasing the number of applied tasks, and the way that both pure mathematics and applications connected with these applied tasks was examined in each content area. In the late 1980s, applied problem solving was utilized more often and in more versatile ways. The increase in application tasks and realistic problems in teaching was also reflected in the fact that students' achievement improved slightly in these kinds of tasks. In 1990, most teachers found it easier to adopt applied problem solving as an important part of their mathematics teaching. (Contains 100 references.) (MKR)

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Pekka Kupari

APPLIED PROBLEM SOLVING IN FINNISH SCHOOL MATHEMATICS EDUCATION IN THE 1980s

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APPLIED PROBLEM SOLVING IN FINNISH SCHOOL MATHEMATICS EDUCATION IN THE 1980s

Results and experiences
in the international context

Pekka Kupari

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Jyväskylä 1994

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ABSTRACT

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The purpose of this study was to examine the role and development of applied problem solving in mathematics education of the Finnish comprehensive school. As a starting point there were, on the one hand, the goals and aspirations set at the beginning of the 1980s for the discussion of the relation between real world and mathematics and, on the other hand, the research-based knowledge of the structure and trends of the theme. The international nature of this theme is well manifested in the fact that a rather strong problem solving -movement transferred to Finnish school mathematics education very soon after it was given a start in the USA in 1980.

The concept of applied problem solving and the descriptive process model constituted the theoretical framework of the study. Applied problem solving was thought as an essential entity of activities in learning mathematics and applications of mathematics, modelling and problem solving often regarded as separate areas, were interpreted as stages/forms of the manifestation of this entity. In the study the author has written a synthesis of the four previous articles which examine the theme at different points of time in the 1980s.

On the basis of the obtained results and experiences, the development during the 1980s in the studying and learning of applied problem solving was positive in many ways. It is true that the changes were cautious and they took place more slowly that had been planned. Many obstacles related to the organization of instruction, to the curriculum and to the teaching culture constrained a faster and more far-reaching progress. We can by no means speak of any 'decade' of problem solving.

At the start of the decade, applied problem solving was intended to be realized mainly by increasing the number of applied tasks, and in the way that both 'pure mathematics' and applications connected with it, were examined in each content area. In the late 1980s, applied problem solving was utilized more often and in more versatile ways.

The increase of application tasks and realistic problems in teaching could also be seen in the fact that students' achievements improved slightly in these tasks. However, the positive side of development was most evident in the change that occurred in teachers' thinking. In 1990, most teachers found it more and more

possible to adopt applied problem solving as an important part of their mathematics teaching. Concerning future development, the essential question really is, in what ways we could promote more integrated ways of introducing applied problem solving as an organic part of school mathematics curriculum and teaching.

Descriptors: mathematics education, curriculum, application, modelling, problem solving, development, comprehensive school

TIIVISTELMÄ

Kupari, P. 1994. Soveltava ongelmanratkaisu suomalaisessa koulumatematiikan opetuksessa 1980-luvulla. Tuloksia ja kokemuksia kansainvälisen kehityksen valossa. Jyväskylä: Jyväskylän yliopisto. Kasvatustieteiden tutkimuslaitos. Julkaisusarja A. Tutkimuksia 58.
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Tutkimuksen tehtävänä oli tarkastella soveltavan ongelmanratkaisun roolia ja kehitystä suomalaisen peruskoulumatematiikan opetuksessa 1980-luvun aikana. Lähtökohtana olivat ne tavoitteet ja pyrkimykset, joita arkitodellisuuden ja matematiikan välisen yhteyden käsittelylle (soveltamiselle ja ongelmanratkaisulle) 80-luvun alussa asetettiin sekä tutkimuskirjallisuuden antama tietämys tutkimusaiheen rakenteesta ja kehitystrendeistä. Tutkimusteeman kansainvälistä luonnetta ilmentää hyvin se, että verraten voimakas ongelmanratkaisun aalto tuli suomalaiseen matematiikan opetukseen hyvin pian sen jälkeen, kun se oli USA:ssa vuonna 1980 pantu liikkeelle.

Soveltavan ongelmanratkaisun käsite ja sitä kuvaava prosessimalli muodostivat tutkimuksen teoreettisen kehyksen. Soveltavaa ongelmanratkaisua pidettiin matematiikan oppimisen olennaisena toimintakokonaisuutena ja useinkin erillisinä koetut matematiikan sovellukset, mallintaminen ja ongelmanratkaisu tulkittiin tämän kokonaisuuden vaiheina tai ilmenemismuotoina. Tutkimuksessa kirjoittaja on tehnyt synteesiä neljästä aikaisemmasta tutkimusartikkelista, jotka käsittelevät teemaa eri vaiheissa 80-lukua.

Tulosten ja kokemusten perusteella soveltavan ongelmanratkaisun opiskelussa ja oppimisessa tapahtunut kehitys oli 80-luvun aikana monin tavoin myönteistä. Tosin muutokset olivat varovaisia ja tapahtuivat paljon hitaammin kuin oli suunniteltu. Nopeampaa ja pidemmälle menevää edistymistä vaikeuttivat monet opetuksen organisointiin, opetussuunnitelmaan ja opetuskulttuuriin liittyvät syyt. Niinpä mistään ongelmanratkaisun vuosikymmenestä ei voi puhua.

Vuosikymmenen alkupuolella soveltavaa ongelmanratkaisua toteutettiin lähinnä soveltavaa tehtävistöä lisäämällä ja siten, että kullakin sisältöalueella käsiteltiin sekä 'puhdasta' matematiikkaa että siihen liittyviä sovelluksia ajan niin salliessa. 80-luvun jälkipuoliskolla pyrittiin soveltavaa ongelmanratkaisua hyödyntämään enemmän ja monipuolisemmin.

Sovellus- ja ongelmatehtävien lisääntyminen opetuksessa näkyi myös oppilaiden oppimistulosten hienoisena parantumisena tämänkaltaisissa tehtävissä. Kuitenkin myönteistä kehitystä voitiin havaita selvimmin opettajien ajattelutavan muutoksessa. Vuonna 1990 useimpien opettajien mielestä soveltavan ongelmanratkaisun ottaminen tärkeäksi osaksi matematiikan opetusta tuntui yhä enemmän mahdolliselta. Tulevan kehityksen kannalta keskeinen kysymys onkin, millä tavoin voidaan

edistää enemmän integroituja tapoja saattaa soveltava ongelmanratkaisu elimelliseksi osaksi koulumatematiikan opetussuunnitelmaa ja opetusta.

Avainsanat: matematiikan opetus, opetussuunnitelma, sovellukset, mallintaminen, ongelmanratkaisu, kehitys, peruskoulu

PREFACE

During the last ten years, considerable change has been taken place in the debate related to the use of mathematics and students' mathematical activities. In most countries there is a vast agreement of the fact that discussion of the relationships between mathematics and real world inevitably will be a part of mathematics education as a whole. Now it is generally accepted that processes of modelling and mathematisation shall be discussed in instruction and instead of standard models and solutions teaching of application, modelling and problem solving skills will be emphasized.

The ongoing progress can be seen as a challenge to the old teaching tradition of mathematics which has been characterized with a rather fixed structure: the repetition of a textbook driven lesson program, the big share of students' silent work with numerous mathematical tasks, and the emphasis of the instant use of symbols. Signs of the occurrence of this tradition could still be observed in the National Assessment of the Comprehensive School (Assessment 90) that was implemented in our country by the Institute for Educational Research in 1990-91.

The Assessment 90 -study was a project designed to assess both the activities and learning outcomes of the comprehensive school. The study took place in the important and interesting phase when the Finnish school system started to move from the old centralized period towards more flexible structures of schooling and school-based curricula. The study focused on several school subjects, and in addition, the comprehensive school was also more extensively assessed as a learning environment.

The purpose of the mathematics substudy was to examine what the state of mathematics teaching and learning is like in the beginning of the 1990s. First results were reported in two collaborative publications in 1993. The present report is the first and more detailed mathematics publication which will provide answers to the question presented above. In the report, the development of the important theme for mathematics education - applied problem solving - is examined during the 1980s and thus it also creates a background for the next publications. Since the theme is of international interest and a topic of lively discussion the report will be published in English.

The Institute and the researchers of this project created the enthusiastic environment for the preparation of the study. Professor Raimo Konttinen and Dr. Pirjo Linnakylä encouraged me in many ways in my work and made invaluable proposals for the improvement of the manuscript. In the writing of the English text I was professionally assisted by Mrs Tuula Granberg, Dr. Sauli Takala and M.A. Liisa Havola. Mrs Raili Puranen and Mr Jouni Sojakka helped me in the word processing. I wish to present my warmest thanks to them all.

Jyväskylä, in August 1994

Pekka Kupari

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1. INTRODUCTION

The importance of mathematics for society in general is widely accepted. This is not because a relatively small group of professionals create interesting mathematical knowledge, but because this knowledge can be applied to many theoretical as well as practical problems. Mathematics is applied in a great variety of areas. It is applied to physics, chemistry, biology, geology, in the health care system and oil exploration. We have mathematics applied to communication theory and risk and optimal search, to sport, interest rates, automobile traffic and military technology. Also the mathematical models of production processes and economics are extremely important.

Today we can find that mathematics has 'hiddenly' structured our social reality and environment in many ways. Mathematical devices which control, organise, predict and manipulate nature and social life have penetrated every part of reality. Our world and surrounding reality is becoming increasingly formalised and mathematised. *Mathematisation* means that mathematical ideas and constructs, either in their theoretical form or in their computer applications, are used to describe, organise, regulate and to foster our human activities. Using the word 'mathematisation' explicitly emphasises that it is we human beings who are putting the mathematisations in place - consciously or unconsciously - and who are affected by them. (Keitel 1993, Davis 1991).

Mathematical modelling is today's high prestige way of describing and predicting. The so-called objectivity of mathematics lends enormous credence and status to its pronouncements. Furthermore, the arcane nature of mathematics - that is to say that mathematics is a domain whose inner workings are hidden or mysterious to the average person - results in the abdication of the processes of social mathematisations to a corps of experts. Although an ordinary citizen is convinced that some engine would not exist without mathematics, for instance, only a few specialists know which mathematical results are hidden in the engine. Keitel (1993) hits the nail on the head when she says that "mathematics itself has become a kind of meta-technology - a technology of technology with the computer as materialised mathematics, as a 'thinking tool'". Mathematical models are not only utilised for description or prediction, but also more and more for prescription. Thus, the process of decision making in our societies has increasingly come under the influence of mathematical modelling. For example, in USA half a billion dollars was spent on the development, use and maintenance of models in 1979 (Davis 1991).

While the social, objective importance of mathematics has dramatically increased, the subjective importance of mathematics taught in schools for the individual has rapidly decreased. More and more explicit mathematical activities which earlier had been necessarily done in specific vocational or general professional areas or daily life activities, are now carried out more rapidly and accurately by machines and technological devices. "Mathematics becomes implicit and invisible; an increasing mathematisation of our society is complemented by an increasing demathematisation of its individual members" (Keitel 1993).

When the development of the student's ability to classify, organise, and model situations that confront him/her in the surrounding world is seen as one main goal of mathematics teaching (Opetushallitus 1991), teaching has many serious challenges to meet in view of what has been said above. Do we have a comprehensive understanding of the direction into which the mathematising of our society is developing? Do citizens need to know, how mathematics is used in our society? Does mathematics education really empower students when they are taking implicit mathematics for granted? What could be the significance of mathematics teaching and learning which is based on applications, modelling and problem solving?

In this publication the role and development of applications, modelling and problem solving in the Finnish comprehensive school mathematics education are examined. The main focus of interest is the 1980s, when the emphasis on problem solving surfaced strongly also in Finland. In the report the author first outlines a theoretical frame to analyze the theme and then describes the international development of the domain during the last ten years. The main research questions are, what kind of position applications, modelling and problem solving have had in the national mathematics curriculum, in which ways the share of these themes could be increased, and how the new emphases were reflected in mathematics teaching and learning in 1990. The report also provides ideas and suggestions as to how applications, modelling and problem solving could be made an essential and established part of mathematics education. The author addresses the research issues by using the data which are included in the four research articles. The articles are papers and reviews which the author presented at international congresses during the 1980s and in the beginning of the 1990s. These papers and reviews have also been published in English in those connections. Thus, the present publication is also a synthesis of the author's earlier research in the field.

2. APPLICATIONS, MODELLING AND PROBLEM SOLVING IN SCHOOL MATHEMATICS CURRICULUM: CONCEPTUAL FRAMEWORK AND INTERNATIONAL PERSPECTIVE

During the past two decades that the Finnish comprehensive school system has been in existence, mathematics education has undergone several different phases of development. In the 1970s, and already in the 1960s, efforts were made to construct the teaching in a way that would help as many students as possible to understand the basic principles and structures of mathematics as well as the mathematical way of thinking (Leino 1984). In the 1980s the principal goal was to present mathematics as a tool, by which a child and youth can learn to interpret the surrounding reality and which is suitable for dealing with the most varied situations. In this connection, the concept of problem solving became central in discussions about mathematics teaching. Some advocated increasing the role of applications in mathematics, while others promoted model-thinking or modelling. However, all these arguments had a common goal: to gain recognition for the basic nature of mathematics and its functional purposes.

The increased interest in the usefulness of mathematics stemmed from the agenda of the mathematics teachers' association in the USA whose first and foremost recommendation was: *"Problem solving (should) be the focus of school mathematics in the 1980s"* (National Council of Teachers of Mathematics 1980). This recommendation was supported by many groups - mathematics teachers, mathematicians, teacher educators, researchers, parents, school administrators - and it was presented, because "curricular materials and instructional processes had lost touch with the nature of mathematics and its uses" (Osborne & Kasten 1992). Emphasising the necessity of applications and problem solving in mathematics education was viewed as theoretical and practical action against the dominance of pure mathematics in school raised by the New Math movement. School achievement studies had also shown that students' learning achievements had remarkably deteriorated, especially in the areas of problem solving and application skills, and more generally also in thinking skills. A good ten years ago, mathematics educators propagating the teaching of applied mathematics represented a unique group. Since then problem solving, applications and modelling have been acquiring increased importance in mathematics teaching as shown by the international discussion and congress activities or by research conducted in the field. The widely spread enthusiasm and activity even gave birth to a series of congresses, especially devoted to this topic.

It did not take long for problem solving to enter also the Finnish mathematics

curriculum. This was partly due to the fact that the process of constructing the comprehensive school mathematics curriculum was currently in progress at the beginning of the 1980s, and the emphases raised by the 'new wave' could thus be instantly included in the mathematics syllabus which came out in 1982 (Kouluhallitus 1982). Another important incentive for debate and activity was offered by the data from the First National Assessment in 1979 which demonstrated that, like in the USA and many other countries, there were great weaknesses in students' application and problem solving skills (Kupari 1983). The shift of emphasis from the core curriculum -thinking which dominated at the end of the 1970s towards the direction of problem solving, indeed created an active discussion in professional publications (e.g., Malinen 1980, Lehtinen 1984, Leino 1984, Haapasalo 1985, Keranto 1985, Kupari 1985, Pehkonen 1985, Heikkilä 1986).

2.1. Theoretical starting points

A massive amount of research has been carried out on mathematical problem solving as well as applications and modelling. Since the late 1970s this has been the topic on which probably more edited monographs, research reports and articles have been published than on any other area in the teaching of mathematics. In this connection it suffices to mention such names as Krulik & Reys (1980), Polya (1981), Burkhardt (1983), Lester & Garofalo (1982), Silver (1985), Schoenfeld (1985), Charles & Silver (1988) and Blum et al. (1989). Without exception, the research has been based on the so-called cognitive learning conception (e.g. constructivism) whose starting point is that when a student acquires new information, he reconstructs it, and the new information becomes assimilated into his already existing knowledge system and helps to transform it. According to this constructivist approach, mathematics is understood as development of thinking processes and as construction of connections, rules, and operational charts, starting from the student's actual experiences (e.g. Underhill 1988).

In the first half of the 1980s the amount of domestic research on the topic was still rather small. The publications and articles were, however, very up-to-date and they dealt with themes like the significance and position of applications and problem solving in mathematics teaching (Lehtinen 1984, Leino 1984, Keranto 1985, Kupari 1985, Pehkonen 1985) and with the methodology of problem-centered mathematics instruction (Haapasalo 1985).

In view of the vast amount of research literature on the topic and the goal and tasks of this study, the author thinks that, in the interest of coherence, it is justified to present a restricted and focused discussion of the theoretical context of the research themes addressed in the present publication. The research by Werner Blum and Mogens Niss, which summarizes well the research domain, has been particularly helpful in the construction of the theoretical framework of the study (e.g. Niss 1987, Niss 1989, Blum & Niss 1989, Blum 1991). This has been a conscious and pragmatically oriented choice.

2.2. The applied problem solving process

The research literature and discussion on applications, modelling and problem solving reveal that the concept formation related to this topic has been rather diffuse (e.g., Lester 1980, Lesh et.al 1984). The concepts that are examined in this study, and their connections, are defined by starting from *the applied problem solving process* and its description (Blum & Niss 1989, 2-3; Schupp 1989; cf. also Haapasalo 1985, Leino 1989). The applied problem solving process is presented in Figure 1.

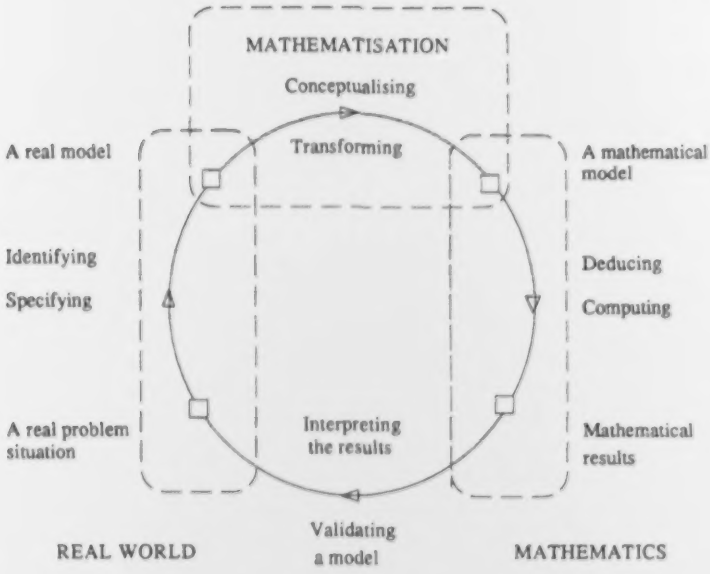

Figure 1. *The applied problem solving process (cf. Schupp 1989).*

A problem refers to an intellectually challenging situation from the point of view of an individual, which contains certain open questions and for whose solution the individual does not have any readily available procedure or strategy available. In the teaching of mathematics, problems are usually divided into two categories. Typical of *an applied problem* is that the situation and problem setting are related to the real world (everyday reality), and in addition it may include some mathematical concepts, methods, and results. The real world, in this connection, means the reality that exists outside mathematics, for example, other school subjects or disciplines, the everyday life or the world that surrounds us. In contrast, *a pure mathematical problem* exists solely in some mathematical environment.

Problem solving is *the whole process* where a solution is sought for the problem. In the following discussion the applied problem solving process is described as specifically related to mathematics teaching, although it can certainly also be seen as a more general model of the problem solving activity.

The starting point for applied problem solving is a *real world situation* (see Figure 1). The first very important phase is that the learner finds the problem, in other words, he becomes conscious of being in a problem situation. The perception of problems is usually also a very demanding phase. The discovery of the problem can be stimulated by a situation that awakens interest and curiosity.

Next, the learner must try to outline, organize, and simplify the problem he/she has discovered on the basis of his/her own prior learning and experience. This phase of the process leads to an *authentic model* of the original real world situation. It is very often necessary to ignore consciously minor disturbing factors or deviations from general regularity: for example, change is considered as linear, movement as constant, air resistance is overlooked, or the globe is regarded as a ball (Leino 1989). At this stage, the learner should be able, among other things, to interpret and transform information from one form to another.

After that, the authentic model must be *mathematised*. For the information, concepts, assumptions, and relations included in the model, correspondences are sought from mathematics (e.g., numbers, letters, expressions, geometrical figures) and the relations between these are represented by mathematical equations, inequalities, functions, conversions (transformations), etc. The mathematisation phase of the model can often be facilitated by suitable questions (e.g., What information has been presented?, What conditions have been given or otherwise prevail?, Are the source information or conditions sufficient?; cf. Haapasalo 1985). The process may be promoted also by bold hypotheses or even guesses made about the kind of model that might be suitable for the situation. As a result of this phase, we get a *mathematical model* that corresponds to the opening situation. The entire process leading from the real world situation to the mathematical model is called *modelling* or *model construction*. In this connection, it should be stressed that this phase of the process is very many-sided; it demands thinking from the students and it can also be taught. In the practice of mathematics teaching, however, the high importance of this modelling phase has not been sufficiently understood and it has been left too much on the responsibility of students.

The mathematical operations that take place within the mathematical model (the right-hand sector of the figure) constitute the most typical activities of school mathematics. It means that computations are performed, reductions are made, and equations are solved, functions are examined and necessary conclusions are drawn from figures, which leads to certain *mathematical results*. The interpretation of the obtained results and conclusions "back" in terms of the real world situation closes the problem solving process. According to Leino (1989), the interpretation helps to explain the significance and meaningfulness of the results, as well as the efficiency and accuracy of the model, and this provides an indication of the usefulness and factuality of the model (validation of the model). The significance of the performed

simplifications may not have been clear until this phase, when it may be necessary to complement the model or even replace it by a new one. The usefulness of the model is thus ultimately weighed only by its functionality in practical reality.

It depends, of course, on the kind and complexity of the given situation, on how often a learner has to go around the process cycle and on how important its single stages are. However, we should never neglect to validate the information we have got by modelling and in this phase students should learn to use non-mathematical arguments, as well.

In addition to realistic and often complex problem situations, there exist simplified and artificial connections between mathematics and the real world. The *direct applications* of mathematics are either standard mathematical models for handling certain mathematical situations or they are mathematical tasks that have been clothed in the verbal form of some other school subject or everyday life. These both types of tasks are generally referred to as *verbal tasks*. They are commonly used in mathematics teaching, although they often give an illusory and even distorted picture of reality. Thus, the term *application of mathematics* in general usage is very broadly applied. In a more restricted sense, it can be used to refer to the activity taking place in the right-hand sector of Figure 1, or also to all the above-described ways by which the real world is brought into contact with mathematics. In this case, also real problem situations can be regarded as *applications* of mathematics.

What has been said above can be summarised in the following way (cf. Stacey & Groves 1989). In general, the term problem solving is used when the tasks and activities are characterised as having a significant degree of novelty and unfamiliarity. It is a matter of applications when the interest lies in the process of applying appropriate mathematics and evaluating the usefulness of the results. Correspondingly, modelling is used to refer to teaching activities where the full modelling process from the formulation of the model to its validation is of interest. In this publication the author will always use the concept *applied problem solving* when some entity of applications, modelling and problem solving is examined, because the situations are mostly of this kind in practice. By using this concept the author wants to emphasise the complexity and process nature of the domain. Only in those cases, in which applications, modelling and problem solving are examined as clearly separate issues, will they be discussed in their own terms.

2.3. Analysis of the process model

2.3.1. *How do we see mathematical problems?*

In general, mathematical tasks and problems are treated in instruction like objects. Tasks are 'given' tasks, with the exception of the few problems the students get a chance to define on their own. The students are then expected to understand the word problem, transform it into a mathematically-tractable form and solve it.

However, in many cases the inbuilt associations seem to lead the student directly to calculations and to results. In this way the students learn to treat the tasks as 'given' ones and everything one needs to know is in it. Bauersfeld (1993) has described in a striking manner problems involved in this kind of approach:

> *"If the individual student's adaptation to the approach favoured by the teacher occurs only across the frequent 'right' or 'wrong' evaluations, and remains restricted to merely a discussion of the technical solution procedures (operations, order, writing schemes), then students will have no chance to develop better strategies or a more sophisticated self-awareness and self-control over their fundamental processes of ascribing and forming mathematical meanings. The technical solution procedures dismiss the tacit but vulnerable production of helpful ideas, and in the end they replace them by the drilled fluency of current solution techniques which are bound to narrow classes of problems and to the specificities of the presentation of the tasks, disappear if not practised regularly, and resist generalisation and any opening towards similarities."*

On the contrary, in the process-oriented integrated view described in the model on pages 5-7 the problems can be viewed as continuous process of problem representation (Bauersfeld 1993). The tasks are not pre-given, but derived from the context, and what counts as relevant is what our common-sense accepts as such. According to the constructivist principles, students develop their own meanings for the symbols, texts and pictures offered by the teacher or the textbook during the solution process. Every step and each decision taken in the process of dealing with the problem changes the issue. Bauersfeld (1993) even suggests that instead of a separate problem we could speak about a biography of this problem related to this specific solver.

2.3.2. *About the difficulty of modelling*

All uses of mathematics, however elementary, involve modelling in the sense that mathematics represents elements in the real world. A major aim of teaching mathematics is to get students to apply their simple mathematics naturally to the kind of situations in which it can be usefully applied. It is understood that this reinforces at the same time their mathematical concepts.

Application situations come about when the students themselves see opportunities for using their well-understood mathematics. It is clear that they are not willing to apply those areas of mathematics which they have not learned properly. Experiences show that students and even teachers invariably choose in modelling tasks such methods and concepts with which they are thoroughly familiar (Ormell 1989). The first phase in getting students to use their mathematics naturally is, of course, to find out where the students are. Can they do this already? Can some one of them? Further on, it is reasonable to ask to what extent the

students recognise the application and modelling situations they meet in the course of everyday life.

There exist different ways to describe modelling process. For example Schupp (1989) has characterised man as a modelling being as follows:

- When we proceed from objects and their connections in the real world to mathematical concepts and their relations we work with images of this real world.
- These images are reductions of the original objects covering only the relevant characteristics.
- This relevance is a feature of the subject who constructs the model.

Many researchers think that it is the formulation phase that is the least understood of the modelling phases. Lambert et.al (1989) have tried to explain certain aspects of mathematical modelling behaviour using the concept of mental models and metacognitive processes. A mental model is a way of representing how individuals construct and utilize their experiences of situations and events. The concept of mental models has increasingly been used in research concerning reasoning about real situations (e.g. Norman 1983; also Haapasalo 1991). A simple framework of cognitive processes in mathematical model formulation is presented in Figure 2.

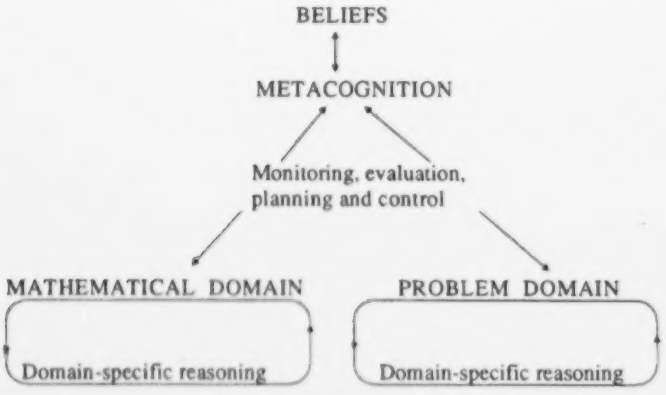

Figure 2. *A descriptive framework of cognitive processes in mathematical model formulation (Lambert et.al 1989).*

Metacognitive processes are responsible for planning, monitoring, evaluation and control of an individual's thinking and reasoning activities. Differences in metacognition could well explain why some individuals with relevant domain knowledge may fail to solve a problem while others with similar knowledge succeed. In addition to the domain specific mental models and metacognitive skills mathematical modelling can be influenced by an individual's beliefs about the domain, mathematics, the environment and their self-image. (Lambert et.al 1989, Schoenfeld 1985).

Lambert et.al (1989) implemented also an experiment with the aim to explore factors related to the modelling attempts. The results of the experiment supported the view that mathematical modelling depends on both mathematical knowledge and domain knowledge but also indicated that the organisation of this knowledge by metacognitive processes also plays a crucial role. The results supported courses and learning tasks which recognise the equal importance of the problem domain and mathematical knowledge and which attempt to demonstrate the general importance and relevance of mathematics in a wide range of situations. This could be a good way to encourage and strengthen a conception of the general applicability of mathematics.

Usiskin (1991) thinks that the process of modelling can be described as an attempt to find mathematical concepts which are isomorphic to situations in the real world. He suggests that students could first consider models which are isomorphic and move gradually to the models which are not. For example, in considering situations modelled by quadratic functions, one might move first from an exact model (e.g. the number of games required for n teams to play each other), to an almost-exact theory-based model (e.g. the path of a kicked or batted ball) and finally to a so-called impressionistic model (e.g. the use of parabola as a model of a frequency distribution).

2.3.3. *How to teach and learn applied problem solving?*

Applied mathematics teaching arouses many hopes and high expectations. However, there exists quite a gap between these high expectations and the fact that only a few investigations have been carried out to check if these hopes can be realized in classroom practice. For the curriculum changes and the development of teaching material reliable research-based knowledge is necessary.

Very many empirical investigations on the applied problem solving are concerned with the question of how to teach skills to apply mathematics. According to Kaiser-Messmer (1989a) these investigations point out that it is necessary to learn to apply mathematics in real world situations - especially with unexpected mathematical methods or varying extra-mathematical context - because most of the students are not able to apply intra-mathematically learned concepts in real world situations. The students' major problems in this process are missing real world knowledge and their haste to use well-known mathematical methods. This is partly

caused by teachers who themselves do not take the world outside mathematics seriously enough and disregard the interpretation phase of the mathematical solution in the real context (Kaiser-Messmer 1989a).

Kaiser-Messmer's own empirical research, which comprised six case studies in the upper secondary mathematics, yielded among others the following results (Kaiser-Messmer 1989a, 1989b):

- The ability to master everyday-life situations is reached by most students. How far this ability is developed depends on the themes of the teaching material and whether they are interesting and important for the students.

- The ability to apply mathematics can be promoted within all students but to different degrees. Application and modelling abilities are attainable in long-term learning processes and it is necessary to discuss the process of applying mathematics (cf. Figure 1) explicitly with the students repeatedly. However, these skills have to be taught and they cannot be expected to develop as a transfer after a pure mathematical treatment. Barriers to the promotion of modelling skills are schematic thinking procedures and sticking to recipes.

- Application-oriented mathematics teaching increases motivation and interest in mathematics of nearly all the students. This strongly depends on the fact how far the theme under consideration coincides with the interest of the students. A short-term increase may take place at once with some small examples. As a long-term development it can be observed that the motivational effects lessen because the students get used to real world examples. The improvement of the students' attitude towards mathematics takes place during a long-term study. It occurs not only with negative-attitude students but also with positive-attitude ones.

2.3.4. *Problems in the assessment of applied problem solving*

One of the most essential questions within the applied problem solving is how to shape and practise assessment in such a way that assessment serves its purposes without destroying the work and activities themselves. Appropriate assessment modes and methods do exist, but for their implementation it is necessary that certain traditional requirements of assessment in mathematics education can be abandoned.

In most educational systems there is a tendency that only areas and topics which are subject to assessment are taken seriously. Thus, if we really want applied problem solving to be properly included in mathematics education it has to subject to assessment in some form or another. Applied problem solving should not be included in the mathematics curriculum just to create a new platform for doing pure mathematics. It should be included in education for its own educational value

and its specific character needs to be expressed as well as possible.

The quality of mathematical modelling cannot be judged on mathematical grounds alone (Niss 1993). That is because excellent models may rely on simple mathematics and very bad models on highly sophisticated mathematics. What matters is the extent to which the resulting model is capable of adequately representing that segment of extra-mathematical world which it is built to represent and the purpose for which it is built. Thus, Niss (1993) wants to emphasise that "the criteria for good mathematical modelling are part of the criteria for good science".

In mathematics teaching we should assess the entire applied problem solving process in all its phases (cf. Figure 1), not just the mathematical parts of it. Assessment should be devoted for example to the following issues (Niss 1993):

- *How far did the students get, relative to their point of departure?*
- *In what way and to what extent was teacher aid or literature assistance used in the process?*
- *Is the correspondence between the issues raised at the beginning of process and the answers given at the end reasonable and are the answers given well justified?*
- *How is the outcome of the work recorded and communicated? Is it possible to follow the investigation undertaken and the arguments provided from the perspective of analysis and critique?*
- *To what extent have the students acquired insights and skills that are transferable to other kinds of situations?*

In which kinds of tasks and by which methods should applied problem solving be assessed? Assessment should take place for such tasks and activities which make room for those content and process components which are characteristic in applied problem solving. Most short term tasks and activities do not do justice to this kind of activity. However, there are a whole lot of tasks and activities suitable for assessment. For instance, extended investigations, student portfolios and projects are appropriate and there is also quite a lot of information and experiences of these assessment modes (e.g. Gillespie et.al 1989, Leino 1992, Abrantes 1993, de Lange 1993, Niss 1993).

Niss (1993) ventures to claim that assessment of applied problem solving is easy. According to him assessment is easy if we have enough courage to abandon certain current requirements that assessment should follow under conditions with time constraints. Most of these requirements are requirements for the general fact that assessment should be objectively reliable. To obtain this it is often considered an acceptable cost for the fact that assessment may not be valid.

For the start, we should accept that assessment of applied problem solving is a complicated judging of a vast variety of components in a complex and often messy problem structure. This implies some subjectivity and disagreement, and it implies that assessment takes time and cannot be standardised. However, it does

not imply that assessment cannot be exercised on a sound foundation of reflection, reasoning and articulated criteria. It also does not imply that assessment cannot be summative, if necessary, and expressed in marks that can be given in ways which are fairly robust to change of assessor. Niss (1993) even asks a challenging question, *"why should mathematics teachers not be able to live with it when teachers of mother tongue composition have lived with that complexity for decades"*.

2.2. State and trends at the end of 1980s

This chapter describes what has been the current state of applied problem solving at the turn of 1990s and which kind of developmental phases have led to this situation. The purpose of this examination is to show that the discussion about the role of applied problem solving in mathematics teaching is no isolated 'fad'. On the contrary, the inclusion of applied problem solving in teaching - although by a variety of names - has, throughout the history of mathematics teaching, been repeatedly introduced and even realized in some curricula from time to time. However, here in Finland this impression of a fad may be more convincing than elsewhere, because the emphatic presentation of these issues in the curriculum plans has been strongly defended by the example of other countries (particularly USA).

Applied problem solving should be part of the mathematics curriculum for the following reasons (Niss 1989, Blum & Niss 1989; also Pollak 1989):

1) To foster among students general *creative* and *problem solving attitudes, activities and competencies* as well as open-mindedness and self-reliance (*the formative argument*).

2) To generate, develop and qualify a *critical potential* in students towards the use (and misuse) of mathematics in extra-mathematical contexts (*the critical competence argument*).

3) To prepare students to being able to *practice applications, modelling and problem solving* in various contexts whether referring to other subjects or occupational contexts (mathematics as a service subject) or to the actual or future everyday lives of students (*the utility argument*).

4) To establish with students a *rich and comprehensive picture of mathematics* in all its facets, as a science and a field of activity in society and culture - and since applications, problem solving and modelling constitute an essential component in such a picture, this component should be allotted an appropriate position in mathematics curriculum (*the picture of mathematics argument*).

5) To assist in *acquiring, learning and keeping mathematical concepts*, notions, methods and results, by providing motivation for and relevance of mathematical studies - such work also contributes to exercise students in thinking mathematically, and in selecting and performing mathematical operations within and outside mathematics (*the promoting of mathematics learning argument*).

Because of its great significance, applied problem solving has in its various forms (see 'definitions', pp. 5-7) been the object of continuous scientific discussion all through the 80s in every world congress on mathematics teaching (International Congress on Mathematical Education). The vast interest focused on these themes has also created new forums, of which the most important one is the International Conference on the Teaching of Mathematical Modelling and Applications. These ICTMA-conferences have been organized biannually since 1983. Publication activity has also been vigorous and abundant. In the description of the state and develop ment of applied problem solving the author utilises the analysis presented by Blum & Niss (1989) which outlines four major trends.

* *The first trend: The arguments for the teaching of applied problem solving have widened*

Traditionally, only two of the above-mentioned five arguments for the teaching of applied problem solving have prevailed: the utility argument and the promoting of mathematics learning argument. The formative argument has been used since the late 60s and it was not until the 80s that the picture of mathematics argument and the critical competence argument emerged. This expansion of the arguments is particularly important, because it is only now that the proper significance of applied problem solving has begun to be seen:

> *"not merely amongst tactical devices to improve the situation for traditional mathematics instruction, but as an integral part of the entity of mathematics education" (Blum & Niss 1989, 7).*

* *The second trend: Increasing globality*

During the 1980s an increased world-wide interest has been observed in applied problem solving, both theoretically and on the practical level. This increased interest has, on the one hand, been geographical and, on the other, it has also been connected with the content of mathematics teaching. Especially, there have been three issues where clear improvement has taken place (Blum & Niss 1989):

a) The number of mathematics curricula and syllabuses including applied problem solving has grown rapidly. This growth could be illustrated by a logistic growth curve, where we are still at the point of rapid growth, but already approaching the section of slower growth.

b) During the period of rapid change in the early 80s, the distance between the activists of this area and the mainstream was very great, when the research and development activities related to applications and modelling as well as the school practice were examined (see also Niss 1987). As regards problem solving, this distance was even greater. Although even today the distance between the front and mainstream is still great, it has nevertheless clearly narrowed down. One evidence of this is the increase of application and modelling materials in the textbooks at all levels of education.

c) Concurrently with the extensive spreading of applied problem solving into the practice of mathematics teaching, the pace of innovation at the front has slowed down. Thus, for example, the selection of application areas or the number of contents to be dealt with in mathematics teaching have not basically expanded since 1984. Neither have the research paradigms changed. The development of the late 80s has meant that the ideas and materials produced in the front are now steadily spreading and becoming part of mathematics teaching. In spite of the fact that the extensive experimentation carried out with these ideas and materials has simultaneously informed us of problems and improvement needs, there has not been equally great pressure and incentive for new innovations.

* *The third trend: Increasing unification*

The conceptual and other linkages between applications, modelling and problem solving used to be rare clear (cf. Figure 1). It is nevertheless possible to distinguish periods of different trends and groupings of mathematics teaching which specially concentrated on applications, modelling, problem solving or the integration of mathematics and other school subjects. At the end of the 1980s these different tendencies started to unify, and it is possible to speak of an increasing unification of the field. The different sectors have broadened their perspectives and become more conscious of and interested in contacts and interaction with each other. This development can be described *"as a transition from a discrete towards a continuous spectrum of interest and activity (Blum & Niss 1989, 9)"*.

In spite of the comparatively international nature of applied problem solving there are national and/or regional differences in the interest and emphasis areas. For example, in the USA, the former Soviet Union, Canada and Israel *the psychological aspects of problem solving* have been extensively researched. In some countries - among others in England, Australia and Finland - remarkable interest

has been shown in *the practical side of problem solving*, for example, in its realisation in the mathematics curriculum for the 6-16 year-olds. The bulk of the research and development work focused on *the theoretical and philosophical aspects of modelling and applications* has been conducted in Germany, Austria, the Netherlands, Denmark, France and India. England has long been the leading country, as regards *the practical solutions of modelling and applications*, while interesting work has also been carried out in the USA, the Netherlands, Australia, Italy and Germany.

Mathematics and physics, both as sciences and school subjects, have always been closely connected, and this close connection has also been reflected in the teaching of mathematics at almost all levels. The connections between the subjects became weaker in many countries during the 80s which was a result of the fact that mathematics opened itself to quite new application areas. Among others, it is considered necessary to demonstrate the field of application possibilities of mathematics more comprehensively than just in physics, so that the teaching would be meaningful and useful for students' life in society. Furthermore, many new application areas provide opportunities of activating mathematics in ways that are more easily comprehensible to the majority of students than in physics.

* *The fourth trend: An extended involvement of computers*

The use of computers has brought new possibilities into mathematics teaching. It relieves mathematics teaching of some tedious activities (e.g. for performing numerical and algebraic calculations or for drawing graphs) and helps in developing new teaching methods. It has been evident for a long time that computers will be an efficient tool in both numerical and graphical treatment of mathematical applications and models. The use of computers does not only mean greater precision and speed in information analysis, but in many cases computers are simply irreplaceable, in order that certain mathematical models can be examined at all.

There exist two somewhat different approaches to increase the use of computers. First of all, there is a trend to quantitatively extend the use of computers in mathematics teaching, particularly when different applications and models are examined. In addition to this natural expansion, another trend has emerged in the last few years aiming to expand qualitatively the utilization of computers in applied problem solving. Currently, there already is available computer software that provides assistance for the problem solving or modelling process or for the application of mathematics to different fields. Moreover, there is software for investigating certain types of problem solving or modelling situations as well as interactive programs for model construction and examination of modelling performances. A very significant characteristic of this new way of using computers is that only a small amount of knowledge about the mathematics included in the situations is necessary.

The use of computers and other technical devices may also entail many kinds

of problems and risks which have already been observed in the classroom instruction (e.g. Blum 1991, Usiskin 1991):

- In most schools it is not possible to have all students at computers all the time. Some problems come out also on tests and examinations because there is a natural tendency to give harder tasks to students when they have access to calculators or computers. Furthermore, working with technology takes time and if the technology does some part of the work it takes quite a bit of time for the students to explain what was done. Also, the management of evaluation with technology is in its infancy.

- The decrease of routine computational and graphical skills and the shift in emphasis towards applications and modelling enabled by computers may make mathematics instruction more demanding for both students and teachers.

- The intellectual efforts of students, necessary in learning, may be replaced by mere button pressing, and new kinds of senseless routines may occur.

- There are also problems in doing homework. If a student has no computer at home, how can assignments be given that require computers? Thus, the students who have a computer equipped with sophisticated software at home are at a considerable advantage over the ones who do not. For example, in USA there is a tendency to favour the use of a calculator when both a calculator and computer can do a job, because students can carry it with them. This also gives the students equal possibilities in their study.

- Mathematics teachers may become interested in computers instead of in application and modelling processes. The computer may also prevent students from reflecting deeply upon mathematical problems by being more engaged in outward activities like constructing technically elegant programmes.

2.5. Applied problem solving in the curriculum

During the past 20 years much of the work in applied problem solving has been devoted to the collection of examples. When, in 1980 the materials available worldwide for teaching applications of mathematics were introduced at the ICME-4, many of them were simply collections of applications. The number of still better and more versatile examples has increasingly grown up and the huge development of computers has influenced this.

However, we can no longer be satisfied with the mere collections of nice application and modelling examples but we have to organise and sequence these learning tasks so that students are able to gain the general ideas and principles they

need to attain the goals of applied problem solving (see p. 13). Because the importance of applied problem solving is generally seen very important at the moment, experience with these ideas must begin early in a child's education and continue throughout the school. Building a curriculum means selection, sequencing and timing of these experiences (Usiskin 1991).

Applied problem solving can be arranged in mathematics curricula in many different ways. One classification of approaches is presented in the following (cf. Lesh et al. 1986, Blum & Niss 1989, Blum 1991):

1. *The separation approach*: Separate courses are produced for applications, modelling and problem solving. In this solution the so-called pure mathematics courses are left outside the applied treatment.

2. *The two-compartment approach*: The mathematics curriculum or syllabus is divided into two parts. The first part is composed of a usual course in pure mathematics, while the second part utilizes the mathematics studied in the first part or some time earlier, by means of applied problem solving.

3. *The islands approach*: The mathematics curriculum is first divided into several parts (for example, content areas), which are then organized by following the two-compartment approach.

4. *The mixing approach*: Learning situations of applied problem solving are used as often as possible, when students are introduced to new mathematical concepts and contents. Conversely, newly acquired mathematical concepts, methods and results are connected, as often as possible, to an application, modelling or problem solving context.

5. *The mathematics curriculum integrated approach*: In this approach problem situations, whether mathematical or practical, are introduced first and the mathematics contained by them is searched and studied afterwards. In principle, the only requirement set for the investigated problem situations is that they lead to mathematics that will create a meaningful and feasible curriculum entity from the learner's point of view.

6. *The interdisciplinary integrated approach*: This approach resembles greatly the previous one, with the only difference that here the close integration of mathematics and extra-mathematical activities takes place within a larger interdisciplinary framework.

The question of a suitable approach, or combination of approaches, for any country or system depends on many factors: for instance, what are the goals and objectives for applied problem solving mathematics teaching and what are the

typical characteristics and specialties of the educational system in question. According to Blum & Niss (1989), the general international picture of the prevailing situation is remarkably unified. In elementary school mathematics (the lower stage of the Finnish comprehensive school) approaches 3 and 4 have been dominating. The situation has been quite similar also in junior and upper secondary school mathematics, although in some instances approach 5 has also been used.

3. RESEARCH QUESTIONS

The purpose of this study is to describe and analyze the teaching and learning of applied problem solving and its development in the Finnish comprehensive school mathematics of the 1980s. The study seeks answers to the following five questions:

1. *How is the national curriculum development in the comprehensive school mathematics related to the international trends?*

2. *What was the position of applications and in what way were applications used on the upper stage of the comprehensive school (especially at the 9th grade) at the beginning of the 1980s?*

3. *What efforts were made to increase applied problem solving in comprehensive school mathematics teaching during the 1980s?*

4. *In what way did the studying and learning of applied problem solving change by the year 1990?*

5. *What factors have constrained applied problem solving from achieving the position they should have in the comprehensive school mathematics education?*

Answers to these questions are reported in the articles presented in the following chapter. The articles constitute the empirical material of the research and therefore they have been included in Chapter 4 in the lightly revised form.

By means of the order in which the articles are presented the author seeks to construct a meaningful entity. It should be noted that, for example, the temporal comparisons in the articles are always associated with the point of time when the articles were written. The first article is a research survey which creates a general picture of the topic by describing the development phases of problem solving, modelling and applications in the curriculum. At the same time it provides material for the examination and comparison of the national and international curriculum development. Originally, the article was prepared for the 1988 Budapest ICME 6 - congress and it was published as an abbreviated version in a monograph on this theme (Kupari 1989b).

The other articles are strongly based on the empirical data and examine more specifically the role of either applications or problem solving in mathematics

teaching. The second article was prepared by the author as a working group presentation for the international congress in Adelaide in 1984 (ICME 5), and was later published in the Yearbook issued by the Institute for Educational Research and the Finnish Association of Mathematics and Science Education Research (Kupari 1985). Articles 3 and 4, in turn, are research papers composed for the 1987 and 1991 ICTMA congresses and they have been published in monographs produced of these congresses (Kupari 1989a, Kupari 1993).

The empirical results of the articles are based on three bodies of research material. The first one is from the year 1979, when the Institute for Educational Research carried out its First National Assessment of the Comprehensive School (Kupari 1980, 1983). The second body of material is made up of the national contribution to the Second International Mathematics Study of IEA from the year 1981 (Kangasniemi 1989). The third body of research material is from the year 1990, when the Second National Assessment of the Comprehensive School (briefly Assessment 90) was implemented. This study was, for certain central parts, a replication of the first assessment study. The nature and content of the research materials will be explained in more detail in the articles.

4. RESULTS AND EXPERIENCES OF APPLIED PROBLEM SOLVING

4.1. Problem solving, modelling and applications in the Finnish school mathematics curriculum 1970-1985[1]

In this article the author discusses the development of mathematics curriculum in the Finnish comprehensive school (grades 1-9, ages 7-16 years), which in the 1970s replaced the binary school system (elementary/secondary school). The presentation concentrates mainly on problem solving and applications, because modelling has not been discussed in the Finnish mathematics curricula as an independent activity. It is, however, mentioned in the curricula and most often in connection with application and problem solving.

4.1.1. *The 1970 curriculum of the comprehensive school*

The report of the Committee of the Comprehensive School Curriculum introduced simultaneously both the new school system and "new mathematics" in 1970. The new curriculum emphasised the development of thinking skills in addition to cognitive skills but the emphasis was not, however, carried out in the details of subject curricula. At the same time the division of school mathematics into three - arithmetics, algebra and geometry - was removed and many teaching contents of algebra and geometry were moved to lower grades.

The curriculum stated the following on application and problem solving (Peruskoulun opetussuunnitelmakomitean mietintö II 1970):

> *"Students are instructed to acquire ways and means as well as methods needed in obtaining and applying mathematical knowledge independently.... For later studies and use of mathematics skills it is important that the student learns to work independently: he/she must get used to obtaining information in books, tables and other sources of mathematical knowledge, and learn to apply the information in practical situations as well as to control his/her own achievement".*

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[1] The paper was prepared as a national review for the theme group 3: "Problem Solving, Modelling and Applications" of the Sixth International Congress on Mathematical Education in summer 1988, Budapest, Hungary.

One aim of the curricular reform was to change the overall structure of mathematics instruction so that the students' mathematical thinking would, through instruction based on understanding, develop continuously from the first grade on. However, the over-emphasised position of set theory and the improper timing of the implementation of the reform only scratched the surface of the intended improvement. New mathematics aroused, at that time, more resentment than acceptance, and according to some experts in the field the reform was leading mathematics instruction in an undesirable direction.

The new school form was probably one of the main causes for insecurity, because the teachers had to acquaint themselves simultaneously with the new school system and the teaching of new mathematics subject matter. In the comprehensive school, the entire age cohort was taught together for two years more than previously. From the 7th grade onwards the students were given an opportunity to study in set groups (the general, intermediate and extensive sets). The effect was also made worse by an administrative change, which broke the old teaching tradition. Class teachers could retain their students for two additional years but at the same time they had to teach subject matter for which they had not been trained. Teachers of the upper level had to take responsibility for the first time over the entire age group covering the whole range of ability, and could have the same students for only three years.

Some experts in this field thought optimistically that new mathematics would expand students' skills in problem solving. From the point of view of students' quantitative thinking problem solving was seen as a key factor, because it was seen to reflect a child's ability to understand the situation of a real problem, take advantage of something learned and apply it. Especially technical higher education expressed the importance of modelling in school mathematics.

In the writings of that period different textbooks and application tasks were perhaps criticized most. The artificiality of tasks and the overemphasis of formulae in the textbooks were criticized. For example, according to Kuuskoski (1974) 'dubious characteristics' had been included in our textbooks:

> *"Very often application tasks include formulae where the numbers are marked mechanically and which have the mathematical sign indicating the operation beside them. When a student is faced with a similar task without the formula he cannot calculate, because the corresponding exercise is missing".*

The appraisals of future curricular improvement considered the development of application and problem solving skills as important aims. Also the national tests of mathematics achievements by the Institute for Educational Research in the early 1970s revealed a need to intensify application instruction. The attainment of the presented aims was considered possible by paying more attention (also in teaching practices) to formal objectives, not only to the presentation of more extensive subject matter.

4.1.2. *Core objectives and basic subject matter*

On the basis of feedback gained from the implementation of the 1970 curriculum, it became evident that the mathematics curriculum was too extensive in relation to the number of lessons available. It was realised that the total number of mathematics lessons was smaller than in both those previous school forms, which replaced the comprehensive school. According to Leino (1984) the change to the comprehensive school system meant that the number of lessons decreased by about one fourth, i.e. by one entire school year of study. Furthermore teachers' experiences revealed that the at that time fashionable "spiral principle" was illsuited to mathematics instruction. The subject matter was presented in bits, there was little time and this resulted in superficial learning.

In 1975 the National Board of General Education (NBGE) set up a group for developing comprehensive school mathematics education. *"A proposal for core objectives and basic subject matter"* was born as a result of the group's work and it was published in 1976. Even as a proposal it clarified the interpretation of the curriculum, which could be seen in textbooks and in the decrease of teachers' insecurity. The proposal presented also some other facts to be considered, one of them being problem solving (Kouluhallitus 1976):

> *"The student should be instructed to observe problems in his environment, which could be solved by mathematics ... A desire to solve problems should be aroused in the student. The problems should be sufficiently interesting, important from the point of student's view and not too difficult in the beginning."*

In different quarters (teachers, textbook publishers, school administration) there existed a fairly unanimous opinion about the necessity of the core objectives and basic subject matter. It was agreed that in this way a more solid basis for students' performance could be obtained, and that the problem of under achievement could be simultaneously diminished.

What did the change then mean in terms of application and problem solving? Concerning the textbooks and use of teaching time there were two kinds of development. On the one hand, after 'the set-theory hysteria' of the early 1970s, teachers returned to the old ways, i.e. they focused their attention to applications and increased the number of tasks requiring applications. As an example of this trend the 8th grade textbooks between 1974 and 1979 included over one third more application tasks (mainly word problems) than before. On the other hand, the decrease in the teaching time of mathematics and the trimming of the mathematics curriculum represented another kind of development. The result was, no doubt, that less time-consuming subject matter was taught, or attempts were made to accelerate the teaching process - the reduction of subject matter being aimed in particular at tasks related to concrete contexts and at the number of applications. The situation raised a dilemma: even though there was a relatively large number of application tasks in the textbooks, they could not be used in instruction (or there was no

interest to use them).

The position of applications in the curricula of the 1970s was analyzed in a study by Leino & Norlamo (1980). The researchers felt that, among others, the following factors were responsible for the underestimation of applications in teaching:

1) teachers' poor knowledge of practical applications,
2) students' unfamiliarity with applications and the fact that they were so detached from real situations,
3) the large amount of time application activities require and
4) the difficulty of applications to many students.

In the same study, vocational school teachers felt that the comprehensive school should increase instruction in applied mathematics (e.g. use of quantities and equations, ratio and proportionality, percentages). Vocational school instruction should especially emphasise contents that are closely connected with real situations and readiness to apply mathematics in practice.

On the basis of comparisons made between different textbooks, vocational school books had plenty of applications, which could be taught in the comprehensive school. In the vocational school textbooks the proportion of purely mathematical tasks was about 30-60 %, whereas in the comprehensive school textbooks the proportion of these tasks was as high as 70-90 %.

Leino and Norlamo (1980) emphasised problem-oriented teaching, where time should be devoted to the proper study of mathematical situations which are interesting and useful to students. Time should also be allocated to discussions on students' own suggestions for solutions. Furthermore, the researchers tried to promote an increase in applications by providing an extensive and rich collection of application tasks in their study. The material was classified according to content area and aimed for use both in the comprehensive and upper secondary schools.

4.1.3. *The 1982 comprehensive school mathematics syllabus*

At the turn of the 1980s, problem solving became the slogan for the next reform. In the new 1982 comprehensive school mathematics syllabus, the goals were set to enhance new ways of instruction that would emphasise creativity and the development of thinking skills. Especially, these new endeavours were introduced in the second and third goals of the syllabus (Kouluhallitus 1982):

"The goal is
- *to familiarise the students in using mathematics for the analysis of their environment and solving problems,*
- *to develop the students' reasoning and evaluation skills and to promote the development of creativity".*

The core syllabus, written for the basis of instructional planning and teaching materials, aimed at reducing the amount of changes required in instruction or material use. That is why the contents remained clearly more traditional than the objectives. The new plan described the syllabus in such a great detail that the generality of the curriculum was no longer criticised by the teachers. The specification and simplification of the national curriculum had reached a certain extreme.

The development of the mathematics curriculum between 1970 and 1985 had three phases presented below. Figure 3 shows the timing of the different phases as well as how they have followed the international (in particular North-American) reforms of mathematics education.

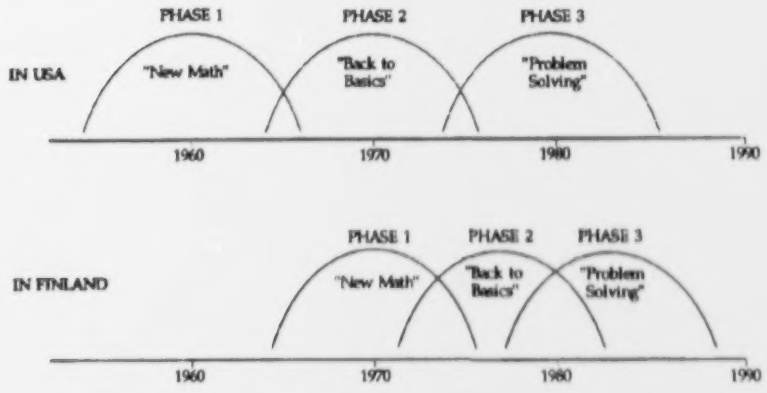

Figure 3. *The developmental phases of the comprehensive school mathematics curriculum in Finland related to the curricular trends in USA.*

The development has many interesting features. One characteristic of interest can be seen in the transmission speed of curriculum influences. On the basis of the figure, the reforms have always reached Finland with a certain delay. With regard to New Math the delay was about 10 years, Back to Basics reached us 5 years later and the emphasis on problem solving was acquired very quickly. Another significant feature has been the fact that in all phases application and problem solving have been present either in the objectives or as aspects to be kept in mind in instruction. Their weight in the curriculum has perhaps been continually increased, but they have not been thoroughly established in teaching practices.

4.1.4. *Some views on problem solving*

Leino (1984) has aptly summarized the development of school mathematics and sketched future trends as follows:

> *"There is a fairly good consensus in industrialized countries about the trends of mathematics instruction. When instruction in the 1960s and the 1970s was planned to be built so that as many students as possible would be able to understand the foundations and structures of mathematics and to learn to think mathematically, the basic aim of the 1980s is to give students a view of mathematics as a means that can be applied to manage a great variety of situations. Some refer in this context to 'problem solving', others to modelling, but the aim is common.... Problem solving, creativity and co-operation should be key words in mathematics instruction."*

In fact, the present decade was to have been the 'decade of problem solving'. Undoubtedly, there has been a serious attempt to move in this direction, which is shown by the new 'phraseology', e.g. "teaching must be developed in a more problem solving direction" (Haapasalo 1985). The real difficulty causing slow progress in thinking and action has been that we simply have had too little knowledge of what is meant by problem solving and problem orientation in mathematics. Since the early 1980s teacher educators and researchers have written quite a lot about the topic and these views have been summarised in the following. The central topics have dealt with the questions of what the significance of problem solving is, what problem solving means, and how problem solving can be taught.

The terms 'problem solving', 'applications' and 'modelling' have been problematic per se. They lack established definitions in Finnish teaching traditions, which is seen, for example, in the fact that in general all word problems are regarded as application tasks or as problems. For this reason, the author has not attempted an unambiguous definition of these concepts here because that would not reflect the real situation in Finland.

There is an agreement about the importance of problem solving and according to Leino (1987) the teaching of problem solving can be justified on the following grounds:

- First, by means of problem solving it is possible to create the skill of being able to use a mathematics repertoire (terms, concepts, computation rules and routines etc.) in a great variety of situations. Leino points out that "many things can also initially be problems in teaching computation rules to young students". The mathematical encoding or modelling of situations is also a typical instance of problem solving.

- Second, problem solving can enhance the acquisition and understanding of mathematics, which means that students learn to act in situations of uncertainty, to structure and model them mathematically. In this connection Haapasalo (1985)

speaks of the possibility of developing individual students' conceptual thinking and inferencing.

- Third, Leino (1987) emphasises affective factors: *"Since problem solving can probably best be taught by making students aware of their approaches to tasks, we are here clearly concerned also with learning to learn and self-awareness and development"*.

By contrast, there are several views of what problem solving means in practise. Most practising teachers' view of problem solving seems to be that "many different word problems should be dealt with in math classes". This means that several sources are used to find a variety of problems, which are worked through in a form of "campaigns" (Haapasalo 1985). Haapasalo thinks that these problem tasks by no means always support the concepts that the syllabus specifies. Furthermore, according to him the teaching of problem solving does *not* mean

- sudden campaigns, 'theme days or years' which are badly integrated with ordinary school work,
- a large collection of problem tasks garnered from a variety of sources to be worked on in class,
- an "enrichment niche" in the math class reserved for practising problem solving,
- the teaching of separate types of problem solving,
- the teaching of separate problem solution strategies in relation to certain problems as if they appeared separately also in practise,
- an attempt to teach students to find a solution to problems as quickly and economically as possible,
- teaching students methods that develop their solution skills only in mathematics.(Haapasalo 1985).

We could talk of roughly two ways of teaching problem solving: traditional solving of problem tasks and the teaching of solution strategies or heuristics. Such heuristics are, for instance, working backwards from objectives, use of special cases, use of intermediate objectives, use of figures. The first way of teaching represents a common method of model learning: the teacher solves a few problems on the board after which the students solve similar problems (e.g., Pehkonen 1986). Leino (1987) suggests that in teaching strategies and heuristics, it is not easy to say which is better: content-focused or process-focused, since both have merits. Research on experts and novices shows that good problem solvers develop a large set of situation and content schemata, which help them to structure problems effectively and then apply some strategy in solving them.

On the other hand, good results have also been obtained by teaching general problem solving strategies. Leino (1987) thinks that the best way is to combine acquisition of new knowledge and mastery of problem solving strategies into a parallelly progressing set of goals. This parallelism means that the problems taken

up should be integrated with other subject matter, so that their practise would be meaningful for students. This requirement is not, however, easy to fulfil.

Practical guidelines given to mathematics teachers have usually been based on the structure presented by Polya (1945). However, Haapasalo (1985) has utilized Galperin's theory of the stagewise development of mental activities as his framework for problem solving process. The stages of the problem solving process are (Haapasalo 1985):

a Orientation to a problem: discovering a problem, defining a problem,
b Working on the problem: specification of the problem, analysis of the problem, finding a solution,
c Solving the problem: carrying out the solutions, defining the solutions, presentation of the solution,
d Interpretation and utilization of the solution: reuse and reorganization of the solution, final review.

Finally, two more things should be mentioned that have been stressed in the teaching of problem solving (e.g., Leino 1987, Haapasalo 1985). It is of crucial importance that the student becomes personally involved in the process of problem solving and, second, that nothing of importance and nothing lasting is learned without the creation of cognitive conflicts and tensions.

4.1.5. *Directions of development after 1985*

The new school laws which came into force in the fall of 1985 emphasise strongly the local right and responsibility for the development of the curriculum and school activities in general. In practice this means, among other things, that the writing of the curriculum itself belongs to the municipality. For this purpose the NBGE stipulated the principles for the comprehensive school curriculum (Kouluhallitus 1985), where the mathematics syllabus again became more general and came closer to the 1970 curriculum. The Cabinet's decision on the allocation of lessons per school year in 1985 had also an effect on the contents of the mathematics syllabus, as the number of lessons during grade 9 was reduced.

From the point of view of mathematics instruction a greater change took place when sets (ability grouping) were removed at the upper level of the comprehensive school (grades 7-9). Sets were replaced by intra-class differentiation, which is supported by the time resource quota system. The new system means that the schools are given a time resource (depending on the size of the school), which the schools can use for the reduction of the number of students per teaching group for certain school subjects. This reduction of the group sizes makes more individualised teaching possible.

Delegation of decision making and the extensive changes in the structures of instruction have opened new and unpredictable situations for the school. Why have

we taken the road of decentralisation and does it mean now that there are quite opposite trends taking place in some countries? Does this have consequences in the position and role of highest school administration? Shall we succeed in developing such solutions in mathematics instruction that make problem solving and applications instructional reality?

During 1981-83 there existed idealistic thoughts to turn the 1980s into the 'decade of problem solving'. Even though this had not progressed much by the year 1985, attempts had been made to proceed to an desirable end. One form of the development activities was to organize theme years. The application of mathematics was the theme in the lower stage of the comprehensive school in the school year 1983-84 and was called Everyday Mathematics. Correspondingly the following year was devoted to geometry. The NBGE stressed the role of geometry in using it in the structuring of the environment and in solving problems. Problem solving was also a special theme in the plan, but only in the late 1980s.

The in-service education of mathematics teachers organized by the NBGE featured problem solving predominantly in 1986-87. Two national seminars on "Problem Solving in Mathematics Instruction" were arranged. The idea of these seminars was to spread new information and stimuli to teacher education as well as practical teaching.

Teachers have usually cited the lack of varied application tasks as the main reason for the paucity of applications in teaching. To improve this situation, the National ICMI Subcommittee decided, early in 1984, to start an applications project in mathematics. The first concrete result of this project was a collection of application tasks adopted from textbooks in several countries. This material, for the comprehensive and upper secondary schools, was published in the autumn of 1988. The project is in progress and during 1989 teacher training colleges will, according to the plan, participate by requiring students to prepare seminar papers and essays on related topics.

Finally, the author wants to mention a national blue-ribbon commission that is expected to contribute substantially to the development of mathematics instruction in Finland. The National Commission on Basic Education in Mathematics and Science, which was set up at the end of 1987, has already provided an evaluation of the current level of general knowledge of mathematics and science in Finland. Furthermore, the commission is expected to prepare a program of activities for 1989-93 for the development of teaching at different levels of education. Undoubtedly, the commission will need to consider the role of applications, problem solving and modelling in the school mathematics of the 1990s and the beginning of the next century.

4.2. Applications in the junior secondary school mathematics[2]

In the teaching of school mathematics applications have an essential role. Applications help to prove the usefulness of mathematics, by means of applications mathematical techniques can be usefully practised, and their use promotes conceptual understanding. The value of applications has always been understood and recognized, but it is generally known that at the present time, the position of applications in school mathematics leaves a lot to be desired. The effect of this can be seen also in learning results. Several recent studies of school achievement at this age level (e.g. Foxman et.al. 1980, Hart 1981, NAEP 1979) have indicated that the application skills of students are deficient and that the application process has become mechanized. These defects were clearly disclosed also by the extensive national assessment of mathematics teaching in the comprehensive school in Finland which was carried out in 1979 (Korhonen & Kupari 1983). As a result, applications have been given special emphasis in the effected curricular reforms and development programs for the improvement of teaching. This was the case also in Finland when the comprehensive school mathematics syllabus was reformed in autumn 1982.

What, then, are the reasons for this deplorable state of applications and how could we bring about a new and more inspiring phase - these are questions to which we should try to find some answers here. In the following, the author tries to provide a couple of themes for the deliberation of these issues. The focus will mainly be on the questions: "How are applications used in teaching at this age level, and what problems emerge from this, in view of the learner's development?" Finally, the author also presents some ideas on what could be done to help the teachers.

4.2.1 *How are applications used in mathematics teaching?*

The word *applications* is used to describe a wide range of different mathematical activities. In his excellent review on applications Burkhardt (1983) gives a good definition of applications:

> *"An application involves the use of mathematics in describing a situation from outside mathematics, usually involving a mathematical model reflecting some aspects of that situation".*

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[2] The abbreviation of this paper was presented on the Fourth International Congress on Mathematics Education in September 1984, Adelaide, Australia.

However, an application is always a relative concept in many ways. Applications can be viewed in relation to the different factors in the teaching-learning process. They can be viewed from the standpoint of: 1) *subject matter*, 2) *teaching practice*, and 3) *the student*. It may be asked, then, to what degree the ambiguity of applications is a consequence of this relativity. Next, these factors are discussed in more detail.

4.2.2. *What is the role of applications in the subject matter?*

We may begin by asking how applications are used in the subject matter. In principle, the position of applications in the learning context is one of these two: in the teaching situation, applications come last, or they are the key idea in teaching new subject matter.

When applications follow the teaching of new subject matter, they are used as a means to illustrate the new mathematical technique. In this case, we speak of *illustrations*, and the teaching method which is used is the traditional text-book centered exposition and exercise approach. Several illustrations of a given technique are usual. For example, linear growth might be illustrated by different shopping and travelling problems and, correspondingly, exponential growth by compound interest and radioactive decay.

When applications provide the central theme of instruction they deal with *real situations* from outside mathematics and the aim is to discuss these situations by means of different models. By way of example, Burkhardt (1983) mentions a study of personal finance which might look at income and expenditure or the use of a mathematical model of capital and interest. The many aspects and viewpoints of such realistic situations make the discussion strategically more demanding for the student, so that the technical demands of the mathematics have to be lower.

Among the major objectives in teaching students of this age is to accustom them to use mathematics in forming an organized perception of their environment and to develop their thinking skills. In the pursuit of these aims applications of real situations play a central role. On the other hand, the significance of illustrations is obvious in the acquisition of firm arithmetic skills, which is one of the main objectives of teaching at the primary level.

In dealing with "real situations" the student must outline and analyze the problem, he/she has to choose the strategy for solving the problem and he/she must form a model and decide on the techniques to reach the solution. In doing so, the student is compelled to rely on his/her own thinking and he/she should be encouraged to do so at all times. The student should be encouraged to explain, argue and defend his/her own thoughts and ideas on solving the problem. Faulty solution efforts should be analyzed together with the student, because they often help in leading the way to the correct solution. If students' thinking processes are maintained in this way, it will give them confidence in their own ability to cope with situations.

Why does the subject matter contain so few realistic applications? There are several reasons for this. First, teachers think that applications of this kind are messy and uncertain involving a wide range of open demands, which makes them unpleasant. In dealing with these problems it is not certain that there is any single right answer. Therefore teachers think that these application problems are not decent mathematics which, in their opinion, is characterized by clarity and precision.

Secondly, teachers believe that realistic applications are too difficult for students. It is true that realistic problems do require a broader view and wider experience and maturity than imitative exercises. But since no signs of the development of these qualities can be seen in the mathematics classrooms it is unreasonable to give the students tasks for which they are not ready at all. It should be remembered, however, that during this transition phase when students change from children to young adults, these qualities should be allowed to develop.

Thirdly, applications of realistic situations demand a lot of time. At least as far as Finnish mathematics teachers are concerned, lack of time is regarded as a central problem, which in turn results from the fact that instruction is strictly tied to the textbook. Since no change has occurred in the amount of teaching contents, although teaching time has been clearly reduced, it follows without saying that unessential and time-consuming material will be eliminated or efforts are made to speed up the teaching process. In Finland, according to Leino (1984) trimming has involved problems related to concrete contexts and applications. When, on the one hand, the increase of applications is an approved development trend (in Finland the number of applications in the textbooks for the 8th school year increased by nearly one third in the years 74-79), but, on the other hand, teachers want to retain the main emphasis of teaching on the practice of arithmetic skills, we drift to a disastrous situation. In order to include applications, the teaching process must be speeded up and this happens at the expense of teaching the fundamentals which are necessary for the applications. Thus the instruction deteriorates even more.

4.2.3. *How to teach applications?*

In this chapter, we will look at applications in relation to teaching practices. In this respect two different approaches can be distinguished.

First, we may speak of *standard models of situations* in the teaching of applications. These are important in building up a "tool kit" of useful models and techniques (Burkhardt 1983). They also provide practice in mathematical techniques and reinforce understanding by providing concrete illustrations of abstract concepts. Standard models are taught didactically by exposition and imitative exercise. It is assumed that, after a sufficient number of illustrations, the student will be able to recognize the characteristics of the standard situation when he meets a similar situation (pattern recognition). An example of this might be calculating the two basic cases of percentages: how much is a given percent of a number and what

percentage is a number of another? However, learning outcomes indicate clearly that some points are easily misremembered or different cases get mixed up. Thus it would seem that by using only standard models it is not possible to assimilate new methods and to use them in practice. The reason for this is, that the repetition of similar situations cannot result in the formation of higher level concepts and generalizations, which is the prerequisite for assimilation of things and which should be aimed at in this age period, according to Piaget and Ausubel (e.g. Bell et.al. 1983, Resnick & Ford 1981).

On the basis of what has been said above, it is obvious that students are able to remember only a small fraction of the models that they employ in the standard applications. Evidence and experience have shown that even small deviations from the standard models of situations confuse students. Thus it is of the highest importance for every student to acquire skill and experience in tackling *new situations*. When a student faces a new situation he or she must resort to his/her own store of knowledge in choosing and, if necessary, in adapting models and techniques. As new situations help the student to distinguish the essential features of solution methods, he/she also learns to make generalizations. The reason why new situations have not been very popular in classrooms follows to a great extent from the tradition of exposition and exercise in teaching where the tasks children are asked to perform involve close imitation of processes demonstrated by the teacher. The role of the teacher is that of a manager, explainer and corrector. It goes without saying that after the instruction students succeed in these imitative exercises much better than in tasks where they have to select mathematics from their own "tool kit" of techniques.

In approaching new situations the didactic method based on exposition and exercise is not appropriate. The teaching methods have to include more open aspects which means that teaching derives from students' existing knowledge base, and that students progress at their own level with the teacher acting more as an adviser and fellow student. In practising the tackling and perception of situations the methods of general problem solving could be applied, for example following the model of Polya (Polya 1945, Resnick & Ford 1981).

4.2.4. *What is the role of applications from the student's viewpoint?*

In the foregoing, applications have been discussed in relation to student's characteristics at this age level. Next, we will deal with one final dimension, in other words, the various ways in which students may experience application problems. According to Burkhardt (1983; see classification below), applications differ as far as *the interest level* of the problems to the students is concerned. There are five types of applications: 1) action problems, 2) believable problems, 3) curious problems, 4) dubious problems and 5) educational problems.

1) *Action problems*, concerning decisions which will affect the student's own life.
Example: Organising one's time to balance the various things one needs or should like to do.

2) *Believable problems* are action problems related to the student's plans for the future or for someone you care about.
Example: "How can I borrow some money most cheaply?" or "Is it worth my studying for two more years, or should I try to get a job?"

3) *Curious problems* are simply fascinating, intellectually, aesthetically, or in some other way.
Example: "Why are total eclipses of the sun rarer than those of the moon?" or "How can sap be 'drawn' up a tree 100 meters high?"

4) *Dubious problems* are intended just to make one practise mathematics.
Example: "Calculate the area of a right-angled triangle when its base is 6 cm and the adjacent acute angle is 45 ."

5) *Educational problems* belong to the category of dubious problems which illuminate some mathematical content or concept so beautifully and enticingly that students want to solve them.
Example: "One drachma was invested at 5 % compound interest in the year 759 BC. To what sum has it grown?"

This is, of course, only one way of rating applications from the student's standpoint. However, it illustrates very well the point how many different kinds of problems there are and, at the same time, how few types of applications are used in classrooms. According to Burkhardt (1983), applied mathematics has rarely aimed further than curious problems and for many students it has been almost entirely discussion of dubious problems. Utilization of this whole range of applications would certainly be significant for the differentiation of teaching, because at this stage of education students' readiness and motivation vary dramatically. We should remember, however, that even educational problems always seem curious to some students and, correspondingly, some students experience believable problems as highly dubious. Therefore, the choice of application problems is of vital importance in mathematics teaching.

4.2.5. *Empirical observations*

In Finnish comprehensive school mathematics teaching, the role of applications is largely as described above. Teachers consider lack of time the main reason for limited use of realistic application situations in teaching. This, in turn, results from

the fact that teaching is too strictly tied to the textbook. Since no change has occurred in the amount of teaching contents, at least not in the direction of trimming, although teaching time has been continuously reduced, it follows without saying that time-consuming material will be eliminated or efforts made to speed up the teaching process. As mentioned before, trimming has concerned problems related to concrete contexts and the number of applications. On the other hand, the emphasizing of applications and the increase in their number has been a recognized and desired development trend during the last ten years. This is illustrated well by the fact that the number of applications in one 8th-grade textbook almost tripled during 1974-79.

In the following, the author examines in greater detail learning results in the area of applications, as compared to other achievements, and the relationships between learning results and teaching. The scrutiny will be based on research material collected in 1979 during the First National Assessment of Mathematics Education in the comprehensive school.

The sample of this extensive situational survey consisted of 2251 9th-graders and 115 teachers from 40 junior secondary schools. Data was collected both on cognitive learning results in mathematics and on structural, attitudinal and process features of the student, the teacher, the home, and the school. The assessment of cognitive achievements focused on measuring objectives defined as most essential (so-called basic objectives and core subject matter; Kouluhallitus 1976). Each student was presented with one out of nine test versions containing together a total of 180 items. Each version had been so constructed that, by using a one-parameter logistic model (the Rasch model), we could produce equated scores for all students. The equated student scores and item parameters also allowed estimations of various subscores, for example, regarding different behaviour categories.

The test items were classified according to Wilson's (1971) behaviour classification as follows: computation, comprehension and application and analysis. The number of items at each level was:

- computation 95 items (53 %)
- comprehension 29 items (16 %)
- application and analysis 56 items (31 %)

In view of teaching, the weighting of the areas was correct in that the dominating role of knowledge and computation was clearly emphasized. On the other hand, the proportions of comprehension and application items did not reflect the true position of these areas in mathematics teaching.

Students' mathematics achievements according to the different levels of behaviour have been described in Table 1. The results are quite surprising. When analyzing earlier learning results, we had observed that a majority of students had very inadequate application and problem solving skills. The perception and analysis of tasks seemed to be especially difficult and therefore students' attempts to solve

problems were almost totally mechanical. So, we were led to expect mastery of application items to be clearly inferior to mastery of computation items. This was, however, not the case, as achievements in all areas were very similar. On the average, students mastered 50 % of the items in different categories of behaviour.

Table 1. *The mathematics achievements of 9th-graders at the different levels of behaviour (estimated means of standard scores (z), standard deviations (s) and α-reliabilities, N=2184).*

Level of behaviour	z	s	rel.
Computation	0.49	0.22	.96
Comprehension	0.53	0.24	.90
Application and analysis	0.48	0.22	.94

Reasons for the uniformity of achievements in different areas may be sought in the items presented and in the nature of teaching. Firstly, many of the application and analyzing items were illustrations i.e. verbally expressed arithmetic operations. Generally they contained only one operation (so-called one-step problems, NAEP 1979), the only additional difficulty compared with "mechanical" items thus being reading comprehension.

The items were now reclassified so that illustrations were placed either in the computation or comprehension category. An attempt was also made at utilizing factor analysis, but the factorizations of individual test versions did not produce viable solutions. The outcome based on reclassification of items is more along the predicted lines (see Table 2).

Table 2. *The mathematics achievements of 9th-graders at the different levels of behaviour after reclassification of items.*

Level of behaviour	z	s	rel.
Computation	0.5[1]	0.22	.97
Comprehension	0.55	0.23	.91
Application and analysis	0.40	0.23	.92

Achievements in the 'improved application area' were about 0.6 standard deviation weaker than those in the area of computation and this difference can already be considered comparatively great. In other words, the mastery of more

realistic application items, which are important from the standpoint of students application skills, was significantly weaker than the first look seemed to indicate (c.f. Table 1). Teachers considered these items a definite part of the core subject matter, although the items of this kind had not, on an average, received very much attention in teaching.

After this, it was interesting to see what impact the weighting of teaching contents had on students achievements. Table 3 shows that teachers put strong emphasis on practical computation skills, although a good half of the teachers often underlined the importance of applying mathematics to everyday situations and of thinking and deduction skills. As for homework (see Table 4), the traditional exposition and exercise came out strongly, as almost all teachers gave homework based on problem types solved in the classroom. Again, about 50 % of teachers often used tasks requiring application of subject matter discussed in the class. Homework containing new situations was very incidental.

Table 3. *Teachers' answers to the question "What aspects do you emphasize in your mathematics teaching?" (N=114)*

Aspect of teaching	Practical computation	Thinking and deduction skills	Application to everyday situations	Application to other subjects
Hardly ever	0.0	1.8	1.8	16.7
Sometimes	20.2	43.9	36.0	64.9
Frequently	79.8	54.3	62.2	18.4

Table 4. *Teachers' answers to the question "How often do you give your students homework of the following type?" (N=114)*

Type of homework tasks	Tasks similar to those solved in the class	Applications of content discussed in the class	Tasks different to those discussed in the class	Individual tasks
Hardly ever	1.8	3.5	41.6	66.4
Sometimes	2.7	46.9	56.6	26.5
Frequently	95.5	49.6	1.8	7.1

Next, the study was focused on students (N=702) whose teachers had in their teaching often emphasized the application of mathematics to everyday situations and had often given these students homework requiring application of subject matter dealt with in the class (= appliers). The teaching in this group was regarded

as having been most application-centred. This student group's achievements in different behaviour categories were then compared with the achievements of other students (= others). It was assumed that the emphasizing of applications would also manifest itself in the form of better performance (see Tables 5 and 6).

Table 5. *The estimated means (z) and standard deviations (s) of student groups in different behaviour categories.*

Behaviour category	Appliers N=(702)		Others (N=1482)		t-test[3]
	z	s	z	s	
Computation (107 items)	0.48	0.23	0.52	0.22	3.91 ***
Comprehension (36 items)	0.53	0.24	0.56	0.23	2.81 **
Application and analysis (37 items)	0.38	0.24	0.41	0.23	2.81 **

Table 6. *The estimated means (z) and standard deviations (s) of student groups in different behaviour categories by set.*

Behaviour category	Extensive course				Intermediate course				General course			
	Appliers N = 249		Others N = 621		Appliers N = 223		Others N = 621		Appliers N = 230		Others N = 240	
	z	s	z	s	z	s	z	s	z	s	z	s
Computation	0.69	0.15	0.69	0.15	0.48	0.17	0.45	0.17	0.26	0.13	0.27	0.13
	n.s.				t = 2.26 *				t = 0.83 n.s.			
Comprehension	0.74	0.15	0.73	0.16	0.52	0.18	0.49	0.17	0.30	0.15	0.31	0.14
	t = 0.85 n.s.				t = 2.22 *				t = 0.75 n.s.			
Application and analysis	0.59	0.19	0.58	0.19	0.36	0.17	0.33	0.16	0.17	0.11	0.18	0.11
	t = 0.70 n.s.				t = 2.36 *				t = 0.98 n.s.			

The hypothesis was not confirmed. On the contrary, the appliers had significantly worse means both in application and analysis and in computation and comprehension than the others. The outcome was unexpected, and it was

[3] *** = $p < 0.01$; ** = $p < 0.1$; * = $p < 0.5$

considered a result of the way students from different sets were divided into these groups. As a matter of fact, the group of appliers did include distinctly more students from the lowest set, i.e. the general course, than other groups. The impact of this factor was eliminated by examining the results by set and this can be seen in Table 6.

We now noticed that there were no differences in the achievements of the groups either in the extensive course or in the general course. In the intermediate course the achievements of appliers were better to an extent which only just reached the level of significance ($p<0.5$). However, we have to bear in mind that the means of appliers were higher in all behaviour categories and not only in application and analysis.

The results we obtained are interesting in several respects. They confirm clearly that application tasks are very much seen in the illustration sense, which favours computation-centred "mechanistic" teaching. Only a limited number of more realistic application tasks were used in teaching because of lack of time, and therefore, their mastery was considerably weaker. Items which are more important in view of learning had received less attention.

Results regarding relationships between the weighting of teaching and learning results reinforce further the picture we have obtained from the teaching of applications. According to teachers' own replies, they emphasize practical computation as well as thinking skills and application of mathematics to everyday situations, but obviously largely subordinated to computation skills. The illustrative and imitative approach in the teaching of applications was manifested e.g. in that when (strong) emphasis was put on applications the results in all behaviour categories improved in the intermediate course. This was not the case in the extensive and in the general course, because there the teaching went either below or above students' "reception level". On the other hand, this outcome seems to suggest that increased utilization of carefully chosen realistic and also new application situations in teaching would improve the mastery of all behaviour categories for all students, but especially the application skills of mathematics.

4.2.6. *What could be done to help the teachers?*

On the basis of what has been said before, there can be no doubt of that clear changes in the teaching of applications are necessary at this age-level. The present-day teaching of applications is characterized by a clear dominance of illustrations, by teaching practice based on the imitation of standard models of situations and by one-sided application tasks. It is obvious that this kind of education cannot promote students' development in the direction of objectives. The responsibility for the change of teaching style lies mainly with the teachers, and therefore we should find means to help them. This is a challenge to which there is no clear-cut answer, but many ideas are sure to be found. Table 7 on the next page presents a proposal for the development areas.

In the table each development area is linked with a certain problem although it must be understood that they are in close interaction with each other. In *teaching materials* there is a special need for multi-level, interesting application material. There is also a shortage of material particularly in the area of easy, interesting application problems relevant to everyday life. Possibilities should also be investigated of utilizing the existing application materials around the world.

We know from experience that it is very difficult to change *teaching practices and styles*. A large number of different development projects have been carried out all over the world, but no major changes have been produced. For example, it has been found that printed material has very little influence on teacher's teaching style (Burkhardt 1983). On the other hand, the success of some projects in the USA, for example, suggests that students have positive attitudes to application-centered teaching and can also cope well with it. Personally, the author would regard this as a very central area.

Table 7. *A model for the development of teaching applications.*

Problem	Development area	
What is the role of applications in the learning content?	Development of teaching materials	Changing
How are applications taught?	Development of teaching styles	external
What role do applications play from the student's point of view?	Development of teaching arrangements	pressures

The development of teaching styles has direct impacts on *teaching arrangements* and some of the emerging questions might be: "What is the optimum size of teaching groups suitable for various open styles of teaching?" or "What arrangements are necessary for the use of microcomputers in the teaching of applications?"

Also various kinds of *external pressures* have an immense impact on education. Up till now, the social pressures, for example, have been for better technical skills rather than the ability to cope with practical situations. We should try to change these pressures coming from various sectors (including the society, parents, school administration, teachers' associations, examinations, media) to make them more positive to application skills.

Finally, we must remember that implementation of changes in the educational system is a very slow process. Therefore we must be prepared for long-term work and failures as well. All efforts must be aimed in the same direction. This implies the necessary support of extensive research and development activities as well as close communication between the various parties concerned.

4.3. Applications in Finnish school mathematics education - research results and development prospects[4]

How do we see the role of applications in Finnish comprehensive school mathematics and what are the development prospects of mathematics instruction? The author discusses these questions on the basis of two recent surveys and Burkhardt's (1983) conceptual frame of reference. Firstly, Burkhardt distinguishes applications as illustrations and real-life situations. Illustrations are used as a means to clarify newly taught arithmetic techniques, and they follow the teaching of new topics. Secondly, Burkhardt makes a distinction between standard situations and new situations. Standard situations usually provide the basis for the teaching of applications. In tackling new situations the student must utilise his or her own cognitive capacity when choosing models and computational techniques in order to reach the solution, and here lies the principal value of these new situations. Through this process the student learns to recognise the essential features of solution methods.

The research evidence that the author presents originates from two investigations. One of them is the First National Assessment of Mathematics Education in 1979 that was carried out both on the lower and the upper levels of the Finnish comprehensive school. Altogether about 1000 students from the sixth grade (12-year-olds) and 100 teachers participated in the study. Similarly from the ninth grade (involving three different sets) about 2200 students and 120 teachers took part (Kupari 1983). Another body of research material was produced in 1981 in connection with the Second International Mathematics Study (IEA SIMS). A total of 4300 seventh graders and 100 teachers were included in this study (Kangasniemi 1983).

4.3.1. *What is the role of applications in teaching?*

In the Finnish interpretation of teaching the term 'application problems' is primarily understood as referring to verbal tasks. Some 80% of the teachers (grade 7 in the IEA SIMS) find it difficult to teach verbal problems (see Figure 4). What it was exactly that the teachers meant by 'difficult' was not disclosed by the other results of this study. However, on the basis of earlier studies it is possible to find several reasons why teachers find it difficult to teach these tasks. An obvious and the most probable reason is the fact that the treatment of application problems (and especially of real application situations) demands a lot of time. Thus, in reality, by the word difficult teachers mean that the task is 'laborious'.

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[4] This paper was presented at the Third International Conference on the Teaching of Mathematical Modelling and Applications in September 1987, Kassel, Germany.

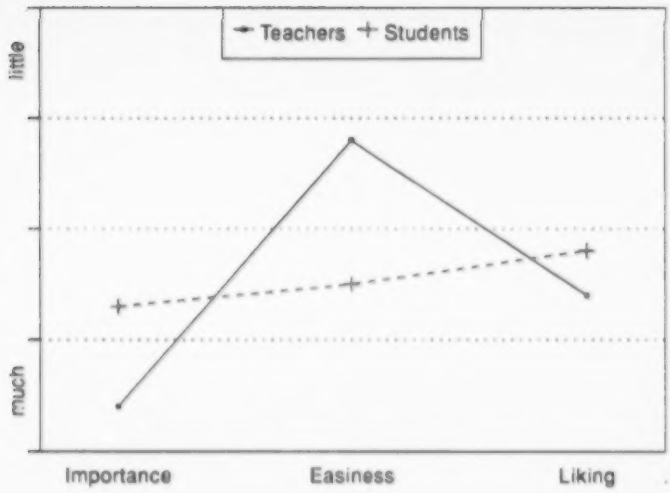

Figure 4. *Teacher and student ratings on the importance, easiness and liking of 'solving word problems'.*

The teachers are unanimous about the importance of verbal problems and about two thirds of the teachers like to teach these problems. The importance of applications can also be seen in the way in which teachers emphasise it in teaching. Teachers put great emphasis on the application of mathematics to everyday situations. Three-quarters of the lower level teachers (grades 4 and 6) and about 62% of the upper level teachers (grade 9) often stress that aspect of application. Application of mathematics to other school subjects, in contrast, is only casually emphasised. In connection with students' homework, about 60% of the 6th grade teachers and half of the ninth grade teachers give their students homework in which they had to apply topics discussed in class.

On the basis of what has been said above, it would seem obvious that in the prevailing Finnish teaching practice teachers do think that it is important to teach applications and, according to their own report, they also realise this idea in their instructions. We might therefore assume that this should somehow be reflected in students' attitudes to applications, concerning the usefulness of mathematics in general, and maybe also in their learning outcomes.

4.3.2. *How do students tackle applications?*

Students at the seventh grade - like their teachers - think that solving verbal tasks is important, but - contrary to their teachers' opinion - they consider it quite easy as well (cf. Figure 4). That problems are considered easy could mean that the great part of application tasks in the teaching materials resemble algorithm tasks, and only differ from these in regard to the wording. In other words, applications are used as a means to illustrate computational techniques. The results may also indicate the fact that students on the upper level (grades 7-9) are capable and willing to work with more demanding and realistic problems.

How, then, do students master applications? Measurable learning outcomes were described as three achievement levels according to Wilson's (1971) classification, namely, in computation, comprehension and application. The results show that student achievements are unvaried. Computation, comprehension and application tasks are mastered very nearly in the same way. The results were the same from the sixth to the ninth grade. Even though the classification of tasks is always problematic, the consistency of the results speak for a real phenomenon. Since the problems which are studied at school (those problems that there is time for) are mainly arithmetic operations clothed in verbal form, it is natural that no great differences occur in their performance level. This interpretation was supported also by the fact that the differences that were found were in items demanding 'real' application. Consequently students had great difficulties in the perception, analysis and modelling of the problems, which led to mechanical solution attempts on the students' part.

No clearly discernible connection was found between the application-oriented teaching style reported by teachers and student performances (Kupari 1985). This research evidence would seem to support the assumption that the teaching of application problems follows the traditional method of expository teaching and imitative practising. After going through enough examples, it is assumed that students are capable of recognising the distinctive features connected with the case when they are confronted by a new situation. A good example of this is the teaching of the different cases of computing percentages (percentage, percentage value). Research evidence and experience have shown, however, that even small digressions from the earlier taught standard cases confuse the students. Through the repetition of similar situations it is not possible also to attain the formation of higher-level concepts and generalisations, which are necessary for more permanent adoption of ideas (Bell et al. 1983; Resnick & Ford 1981).

4.3.3. *On the development possibilities in teaching applications*

It seems that current teaching of applications is characterised by a clear overemphasis of illustrations, by teaching practice based on the imitation of standard situations ('formula calculation') and by the one-sidedness of the problems.

While teachers consider the teaching of applications to be important, and say that they stress the use of applications in their instruction, this does not in reality signify any changes in the teaching practices nor that students develop in the direction of the objectives set for instruction. The greatest problem is the scarcity of teaching time in relation to the subject matter to be taught. It is true nevertheless that the responsibility for any change in teaching remains with teachers. Therefore, we should find ways to help the teachers.

In the fall of 1985 a new school legislation came into effect in Finland. Along with the legislation the Principles of the Comprehensive School Curriculum were put into use, in which the educational objectives of mathematics instruction are identical to those of the 1982 curriculum with its emphasis on application and problem-solving. The objectives are up-to-date in this respect. When, however, the subject matters of the syllabus are more conservative than the objectives, an obvious conflict arises manifesting itself as insufficient teaching time and as a neglect of the new objectives.

The school laws also meant that at the upper level of the comprehensive school (in foreign languages and mathematics) the system of setting (ability grouping) was abandoned. Teaching is now based on a syllabus common to all students. Local authorities and schools can direct their educational resources according to local needs. The most important thing in this reform is that the new system has made it possible to cut down the size of teaching groups (from 23 students to 16). In addition, new teaching groups can be formed in a flexible way, with one or two teachers working with one student group, which should give good opportunities to study real application problems. It is of vital importance that this reform of the teaching arrangements should promote more versatile teaching methods.

Experience has shown that it is very slow and difficult to change teaching methods and styles. For example, printed materials have been found to exert only marginal influence on teachers' styles of teaching (Burkhardt 1983). On the other hand, it is imperative that teaching methods should become more varied. Great pressures and expectations are directed towards teachers' basic and in-service training. As first steps in the direction of change might be considered those problem-solving seminars, which have been organized for comprehensive school mathematics teachers in the two past years.

4.4. Changes in the teaching and learning of applications in Finnish comprehensive school mathematics 1979-1990[5]

In the early 1980s, special emphasis in Finnish mathematics teaching was placed on applications and problem solving. The present paper examines the implications that this change has had for teaching practices in the 9th grade of the comprehensive school. This is done by comparing the results of two assessment studies. In addition, the paper introduces one method of evaluating the changes that have taken place in students' learning achievements and discusses problems connected with the evaluation. The discussions are based on a modified Tyler model (see e.g. Ljung & Pettersson 1990), where mathematics education is seen as an interaction process between the individuals and their environments. The term *application* is generally used in the sense presented by Niss (1989): an application of mathematics is a result for the process of applying mathematics to any extra-mathematical area. In some sections, however, we also speak of *applied mathematics* which then refers to one of the content areas in the mathematics curriculum. In some contexts we also use the combined expression *application and problem solving*, because the curriculum uses it, and because these areas are partly seen as overlapping (see Niss 1988).

4.4.1. *The importance of applications and problem solving in the curriculum and in teachers' in-service training*

At the beginning of the 1980s the new comprehensive school mathematics syllabus in Finland was constructed in the spirit of 'applications and problem solving'. The new emphasis was also very well manifested in the goals of mathematics teaching (Kouluhallitus 1985). On the other hand however, the aim was to proceed so that no major changes would need to be made in teaching and teaching materials. On the basis of the previous experiences (e.g. during the New Math -movement), it was not considered appropriate to disturb teachers' practical teaching work in schools too much.

In the mathematics textbooks of the late 80s the emphasizing of problem solving and applications appeared clearly in increasing of these kinds of tasks. The problem remained, however, how and when these tasks should be taken up in teaching. Many teachers felt that the tasks in the textbooks were too one-sided and demanding, and thus suitable only for some of the students. The solving of these tasks was left for better students only, to be done at the end of lessons, because

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[5] The paper was presented at the Fifth International Conference on the Teaching Mathematical Modelling and Applications in September 1991, Noordwijkerhout, The Netherlands.

simpler, more interesting tasks, suitable for weaker students, were lacking. This student differentiation problem, which is closely connected by teachers with shortage of time, become even more acute in the late 80s. In 1985, the sets were abandoned at the upper stage of the comprehensive school (in mathematics and foreign languages), and since then students have been taught in mixed-ability groups of about 16-19 students.

At the beginning of the 1980s, 'theme years' or 'theme days' were used to make the objectives of the new syllabus familiar to mathematics teachers. In teachers' in-service training, the field of applications and problem solving has been discussed in the late 80s. During the years 1986-87, two nation-wide seminars were organized by the National Board of General Education under the heading "Problem solving in the teaching of mathematics". Approximately 40 mathematics teachers and teacher trainers participated in the seminars. The seminar activity continued in 1989-90, under the title "Applications in school mathematics", mainly with the same participants as in the earlier seminars. The application seminars formed a unity focusing on the process of change which had occurred in England (cf. Mathematics counts 1982). Led by Professor John Berry, the participants studied learning situations which could be used to make mathematics instruction more varied and learning more meaningful. At the same time, the participants became acquainted with the policy through which the change had been promoted in England.

The development described above shows that curriculum reform and teachers' in-service training were the means by which we sought to improve the skills of using mathematics in the 1980s in Finland. The in-service training activities were based on the idea that the participating teachers and teacher-trainers would continue to spread and pass on the training they had received, in their own working environments in different parts of the country. The possibilities for training at the local level had also improved, for according to the 1985 School Act, municipalities were given more freedom to carry out their in-service training. For example, the number of separate in-service training events organized by the Continuing Education Centers of universities increased rapidly, and in addition, municipalities made long-term training contracts with these centers. This paper tries to find out what has happened to the practice of mathematics teaching and possibly also to the students' learning results during 11 years. The author presents three problems:

- **The first problem** concerns the understanding of mathematics applications and the changes that have occurred in it. It was interesting to know if applications had the same 'content' in 1990 as in 1979. Had mathematics teachers' conceptions of applications changed in the course of time?

- **The second problem** deals with the changes in teaching. What kind of changes had taken place in mathematics education as a whole? How had the teaching practices changed? Did applications and problem solving now have more weight, as compared with computational skills, than before? Do teachers favour

applications as homework more in 1990 than they did in 1979?

- **The third problem** addresses changes in the learning of mathematics. In what way and how reliably can we compare learning achievements measured at different times? What kind of changes could be discerned in students' learning achievements?

4.4.2. *Methods*

Background. For the past 15 years or so, the Institute for Educational Research of the Jyväskylä University has carried out national assessments of the Finnish comprehensive school, both as regards the different school subjects, and also in a more general way. The first assessment study was carried out in spring 1979, with four or five separate projects participating in it, including a mathematics project. The second corresponding study was implemented in spring 1990, including eight to nine other research projects, in addition to mathematics. In mathematics, the purpose of the assessment studies has been to evaluate teaching and student achievements at three grade levels of the comprehensive school (fourth, sixth and ninth grades).

Samples and measurement instruments. In 1979, 2251 students and 114 teachers participated in the first study. The corresponding figures in the 1990 study were 1164 students and 67 teachers. The smaller numbers of participants in the 1990 study were due to the fact that the sample size was on purpose cut down, in order to reduce the costs. The sample size of the first study was also increased by the fact that, at that time, there were three sets (ability groups) in the 9th grade, and we wanted to include a representative sample of each of them. It should be pointed out that the teachers who are examined here are teachers who have become included in the study through the representative student sample. Thus it is not a representative sample of all mathematics teachers of the upper stage of the comprehensive school in Finland. In practice, however, it does represent this population quite well.

The shares of girls and boys in the samples were almost exactly the same in 1990 as in 1979. There were a little more boys (53 %) than girls (48 %). Noticeable changes had taken place in students' mathematics grades between the studies. The number of poor and satisfactory grades had decreased and, correspondingly, the number of good and excellent grades had increased. When more than one third of the students received good grades 8-10 (on a 4-10 scale) in 1979, now more than half of the students got these grades. The use of grade 9 had increased most. Great changes had also taken place in the use of calculators, when compared with the year 1979. Now more than half of the students reported that calculators were often used in mathematics lessons, whereas only about 13 % had been of this opinion in 1979. Furthermore, about 30 % of the students now reported that computers had

been used in mathematics instruction.

Considerable changes had occurred in the group of teachers. The number of female teachers had grown nearly 10 %-units from 1979, and now the shares of female and male teachers were nearly equal (women 47.9 %, men 52.1 %). The teaching experience of mathematics teachers had also grown. Whereas in 1979 the length of teaching experience was, on average, 8.8 years, in 1990 it was 13.8 years. The experience in comprehensive school teaching had, of course, grown a lot, because 10 years had passed since the introduction of the comprehensive school system. In 1990 about 90 % of the mathematics teachers were formally qualified. This was a very significant change, because in 1979 the share of formally qualified teachers was only 65 %.

In both studies the data were collected by means of mathematics tests and different kinds of questionnaires. In the 1990 study a total of 99 mathematical tasks were presented to the students in five test booklets. Each test booklet contained a common core of 10 items and each student answered only the items in one test booklet. The test booklets contained 53 items that had been included in the 1979 study. By means of a student questionnaire, information was gathered on their background, studying practices, and their attitudes towards mathematics and teaching. Correspondingly, by means of a teacher questionnaire, information was collected on the teachers' background, their opinions of the changes in mathematics education and practices in mathematics teaching.

Behavioral classification of contents. The behavioral classification of mathematics items was in both studies performed by using the modified Wilson's (1971) taxonomy. Although the content validity of this behavioral classification has been regarded as satisfactory, its structure and usability have also been much criticized (e.g. Ormell 1974, de Landsheere 1979). The behavioral classification was used, following the Second International Mathematics Study (IEA SIMS), to describe the objectives of content areas. This way it was possible to establish the "topography" of the curriculum grid (Travers & Westbury 1989), on the basis of which the mathematics tests could be constructed. The other argument for the use of the classification was connected with the nature of the sampling method used in the study. Because the sampling units were teaching groups, and not individual students, it was possible to use the behaviour classification of content areas to describe and illustrate the level at which the students in the teaching group were expected to be able to function. In both studies, a well-experienced group of teachers classified the research items into three levels of behaviour, in other words, into items of *computation, comprehension* and *application*.

4.4.3. *Results*

Teachers' conceptions of applications

We examined the first problem by analyzing the teachers' assessments of the items of *applied mathematics*. In the 9th grade, the objectives in this area put strong emphasis on the use of mathematical knowledge and skills in verbal application tasks (Kouluhallitus 1985). Furthermore, the concepts of ratio, proportionality and percentage, which are important in the use of mathematics, are strengthened and deepened. Also, the study of statistics is included in the objectives.

The assessed item group of applied mathematics contained a total of 65 items, most of which were taken from the 1979 study, plus a few items from the IEA-study in 1981. When teachers in 1979 and 1981 assessed the behavioral level of the items, 63 % of the items were classified as being at the level of application.

In spring 1990, the result of teachers' assessment was unexpectedly different. In the teachers' opinion, only 11 % of the items were of the application level. The change from the situation 11 years earlier was really significant. The result clearly indicates that teachers' own criteria of application behaviour had changed. Mathematics teachers no longer regarded a task as an application task, simply because it was in a verbal form. Now they probably based their assessment more on what the task demands of the student. It may be assumed that this change has partly been the result of in-service training and active discussions during the 80s.

Changes in the teaching of applications and problem solving

The results above illustrate the change that could be seen in teachers' way of thinking and their conceptions of application. But did this change also affect the teaching practices? We could investigate this part of the second problem by looking at the results of the teacher questionnaire.

According to the results, a large proportion of the teachers (40 %) felt that the teaching of mathematics had become more difficult and demanding with the teaching of mixed-ability teaching groups. The change could be seen in that whole group instruction had decreased, and the need for individual instruction, in contrast, had increased. At the same time, according to the teachers, teaching had become simpler and more superficial, when neither resources nor means were sufficient for varied differentiation. In the teachers' opinion, both high and low achievers suffered from the prevailing situation, and it was also claimed that as a result the standard of learning in mathematics had gone down. However, some of the teachers (approx. 5 - 10 %) found positive aspects in the change, as well: the working climate had improved, student activity had increased and weaker students had improved their achievements.

We get a better picture of the importance of applications and problem solving in the realization of mathematics instruction and of the changes in it, when we

examine the answers given by teachers to two more questions in the teacher questionnaire. In the first question we asked "How often do you emphasize the following aspects of school mathematics in your instruction: computational skills, thinking skills, application skills and problem solving skills"? The classification tries to describe the dimensions used by teachers when they are speaking about the nature of school mathematics. The dimensions overlap to a great extent; for example, thinking skills, application skills and problem solving skills are closely tied together.

The results indicated that the weight of computational skills had decreased in the teaching of mathematics, but not very much. Teachers reported that they emphasised thinking skills more now than in 1979. Application skills, instead, were stressed somewhat less than in 1979, probably because problem solving was rather heavily emphasised as an area of its own. In 1979 no questions were presented regarding the share of problem solving in teaching, because at that time the 'core curriculum' and 'back to basics' - thinking were still dominating and problem solving was seen as a very unrealistic objective of teaching.

The second question covered students' homework: "How often do you give your students the following types of homework?". We were especially interested in two kinds of homework: a) homework in which students practise the skills taught in the previous lesson, and b) homework in which students can apply mathematics discussed in the class. The results showed that teachers continued to emphasize the teaching of computational skills and methods, and that students' homework regularly included a lot of this kind of practice. In addition to these, teachers had introduced more application and problem solving tasks. We can see at least two reasons for this change. Firstly, these kinds of tasks have been added to the teaching materials. Secondly, as a result of the in-service training and the discussion that has taken place, teachers have begun to see the importance of the skills of using mathematics. In this way the goals and ideas presented in the curriculum plan at the beginning of the 1980s are gradually taking the form of practical steps.

About the changes in learning achievements

Students' learning achievements are always of great interest to the school community and also to the general public. Questions related to them seem sensible in this situation, especially since the interval between the measurements is 11 years, and it seems somehow realistic to expect changes in learning achievements during that period.

The questions posed are, however, very problematic at least for two reasons. The first problem is how to analyze change in the case of cross-sectional studies. The second, more specific problem is whether the mathematics items or item groups are comparable when we examine them in the context of their own time. We already discussed the question above in terms of rating the items. More information can be obtained, for example, by analyzing the results by means of the Item

Response Theory (Rasch model, see e.g. Lord 1980). However, in this presentation we do not pursue this further. Instead, in the following we describe a method of analyzing changes in mathematics achievements between 1979 and 1990.

Seven items of applied mathematics that had been presented to students in both 1979 and 1990 were chosen for analysis. In their assessments the mathematics teachers placed all the items, except one item, at the application level. Changes in student achievements regarding the item group were analyzed by using log-linear models (see e.g. Christensen 1990). By means of the adopted model, it was possible to analyze simultaneously all the associations between the included variables and the results were easy to interpret (see Table 8).

Table 8. *The proportional indices expressing the achievement changes over time by items and within different student quartiles (Malin & Kupari 1991)*

	Student quartiles			
Items	I (weakest)	II	III	IV (best)
1	11.0	2.4	2.4	1*
2	11.0	1	1	1
3	1.9	10.7	3.5	8.2
4	1	3.5	3.5	8.2
5	1	1	1	4.5
6	1	1	1	1
7	1	2.3	1	1

* unities (1) mean that there was no difference in achievements or the difference was not statistically significant

According to the results, 9th-graders of the comprehensive school were better or at least equally good in 1990 as in 1979 in solving these items of applied mathematics. The improvement in achievements depended much on the item content and also on students' success in the whole measurement. The latter was used to divide students into four quartiles. One interesting observation was that the students in the weakest quartile improved their achievements most in difficult items (items 1 and 2), whereas the students in the best or intermediate quartiles progressed most in easier items (e.g. items 3 and 4).

For example, in the three percentage items the changes between the two studies were highly different and even surprising. The 'basic' percentage problem (item 6: an application of 'How much is a % of b?') was among the seven analyzed items the only that was in 1990 mastered slightly less well than in 1979 (p79 = 42; p90 = 36). In the two-stage problem (item 3), where the student first had to subtract

and then calculate 50 % of the difference, an improvement took place in the achievements of all quartiles compared to 1979. The improvement was remarkable in the second and highest quartiles. The average improvement in achievements was greatest (18 %-units) in the third percentage item (item 4), where the student had to calculate the reduced price of a product when the original price and reduction-% were known. The highest quartile students clearly improved their achievements, in the two intermediate quartiles also to some extent, but the achievements of the lowest quartile remained the same.

In 1979, the differences between the lowest and highest quartile (i.e. the general set and the extensive set) were considerable in the two really difficult items. In 1990, these differences narrowed down significantly, because the improvement in the weakest quartile was so clear. On the other hand, in the three easier items the differences between the best and the weakest student quartiles multiplied.

4.4.4. *Conclusions*

In the 80s, significant changes took place in Finnish school mathematics. The aims of the curricula constructed at the beginning of the decade laid strong emphasis on problem solving and application, as was the spirit of the times. In the nationwide and local in-service training of mathematics teachers, application and problem-solving were the main themes in the whole country especially in the late 80s. The education policy decisions made in 1985 had contradictory effects. The abandonment of sets at the upper level of the comprehensive school and the heterogeneity of teaching groups meant that teachers had to get accustomed to a new situation. Although teaching groups in mathematics were reduced considerably in size, teachers had to cope with more difficult problems of differentiation than before. On the other hand, increase in decision power at the local level also offered schools and teachers new possibilities for the organization of teaching. The research data collected in 1979 and 1990 give interesting information on the share of application and problem-solving in the teaching of upper secondary school mathematics, and on the changes that occurred between 1979 and 1990.

In 1990, mathematics teachers had different conceptions about applications than 11 years earlier. This was manifested clearly in the way teachers at both times assessed the items used in the measurement. Teachers had stricter criteria for application tasks in 1990 than in 1979: now an item has to meet also other criteria apart from the one that it is in verbal form. This change in teachers' way of thinking was probably partly influenced by the educational activity and discussions regarding application and problem-solving in the 80s.

Teachers' opinions about changes that had taken place in mathematics teaching as a result of the above education policy decisions were a clear indication that teaching had become more difficult and strenuous. Teachers thought that teaching also had narrowed down and become more superficial, because means of differentiation were inadequate. Furthermore, a majority of teachers supposed that

the standard of mathematics achievement had gone down. The share of application and problem-solving in teaching had increased to some extent, although the practising of computation skills was emphasized almost as much in 1990 as in 1979. The increased share of applications was manifested especially in that teachers now fairly regularly gave application tasks as homework. This, in turn, was a result of an increase in the number of these tasks in textbooks.

From the point of view of educational policy, an item-based analysis of applied mathematics yielded interesting results about changes that have taken place in learning achievements. According to these results

- the abandonment of sets does not mean weaker achievements or a lower standard in mathematics,
- in mixed-ability teaching groups weaker students have more varied opportunities for learning and students are also able to learn new things - also the best students should be offered tasks corresponding to their learning potentials.

On the whole, the development in the 80s seems to have had a positive impact on the role of applications in school mathematics. On the other hand, the changes have been cautious and they have taken place more slowly than planned. Local and school-based development of teaching, which is crucial from the point of view of the reform of teaching, has not started in the way hoped for and in the present economic situation it may take years before it can be revived again.

5. SUMMARY AND DISCUSSION

In the above the author has described the role and development of applications, modelling and problem solving during the 1980s in the Finnish comprehensive school mathematics education. This description has drawn on the four research articles published in international contexts. The articles deal with the topic at different times, and also from somewhat different perspectives, and they have been presented in Chapter 4.

The findings presented in the articles are summarized in this chapter. The summary follows the research questions (p. 20) and utilizes the conceptual framework and the wider international development of the domain presented in Chapter 2. Through the model of the applied problem solving process the author has tried to demonstrate that mathematical applications, modelling and problem solving, which often have regarded as separate activities, are in fact stages or aspects of a certain integrated unit of activity and that they thus constitute an essential area in mathematics learning. In addition, some developmental prospects of applied problem solving in the curriculum of school mathematics are examined on the basis of the summary.

5.1. How is the national curriculum development in comprehensive school mathematics related to the international trends?

To make the relationship between Finnish and international curriculum development more comprehensible, it is necessary to examine it for a longer period than just the last decade. The transfer to the comprehensive school system in 1970 provides a natural starting point for the examination of the development. At the same time, each sub-theme (applications, modelling, problem solving) and its manifestation and emphasis in this development process can be better described. This development was described in Chapter 4.1.

Since 1970 there have been three distinct phases in the development of the comprehensive school mathematics curriculum. Curriculum changes have always tended to follow international, mainly American, reform trends. *The first phase* took place at the beginning of the 1970s, when both mathematics teaching and the whole school system were reformed in Finland (*the New Math - phase*). The aim of the teaching reform was that, through instruction based on understanding, students' mathematical thinking would develop smoothly, starting from the first grade. Some optimistic opinions were also expressed claiming that the new math teaching

would improve students' skills in problem solving. From the point of view of students' quantitative thinking, problem solving was considered to be in a key position, because it reflects a child's ability to understand the situation of a real problem, to utilize and apply what he has already learned. Authorities especially in the sphere of technological higher education emphasized the importance of model thinking in school mathematics.

Textbooks and application tasks were the focus of most severe criticism and especially the artificiality of tasks and the way textbooks were bound to certain formalism was criticized. In curriculum development the advancement of problem solving and application skills were considered important objectives. It was believed that these objectives could be achieved if, instead of presenting a great amount of subject matter, more emphasis were put on concept formation, deduction and thinking skills. However, this did not come true in practice.

The experiences and results of the early 1970s revealed that the mathematics curriculum was quite broad and too large in relation to the number of available lessons, which had been reduced by about one fourth when transferring to the comprehensive school. The 'spiral principle' was also conveyed imperfectly by the curriculum: the mere chopping of the subject matter into splinters led to very superficial learning. It was attempted to remedy these defects during 1975-76 by defining the basic objectives and basic subject matter of mathematics education. This was a counter-reaction to the new mathematics: after " the set theory hysteria" steps were taken to return to the old practices (*the Back to Basics - phase*).

As regards applied problem solving, this *second phase of curriculum development* had two kinds of consequences in the latter half of the 1970s. On the one hand, this was manifested by an emphasis on application and by even threefold increase of application tasks in some textbooks. On the other hand, the trimming of the curriculum connected with the definition of the basic objectives and basic subject matter meant quite different type of development in teaching practice. Time-consuming material was either cut down or attempts were made to speed up the teaching process, so that the trimming particularly involved tasks related to concrete contexts and the number of applications. The situation was controversial: although there were more application tasks available in the textbooks, it was not possible to deal with them in instruction.

At the end of the 1970s, it was increasingly emphasized that teaching should be more problem-centered and that time should be devoted to the unhurried examination of mathematical situations that are useful and interesting to the students, and to the discussion of the solutions proposed by students. Practical steps were also taken in this direction by publishing some compilations of application tasks for the comprehensive school and upper secondary school.

The beginning of the 80s meant the start of *the third phase* in the development of the curriculum, when the agenda of the NCTM raised problem solving to a key position in mathematics teaching (*the Problem Solving - phase*). Even this rise of problem solving could be understood as a counter-reaction - this time to the excessive return towards the basics.

In Finland, the new mathematics syllabus was constructed in 1980-81, placing emphasis on problem solving and application, which was clearly shown by the teaching objectives. Effects could also be seen on the teaching materials, where the number of applications and different problems increased. Many teachers thought, however, that the problems of the textbooks were too one-sided and demanding and that they were suitable only for part of the students. Since there was a shortage of tasks suitable for weaker students, the treatment of application and problem solving tasks was postponed to the end of the lesson and left only for the better students.

In 1985, the school legislation was reformed. In this connection, the National Board of Education also issued the new principles for the curriculum. These decrees had many kinds of impacts both on the content and practical realization of the curriculum. The mathematics syllabus became broader again and approached the style of the 1970 curriculum. On the upper stage of the comprehensive school teaching contents had to be cut down, because the number of mathematics lessons per week was reduced by one. A more significant change, as regards teaching, was the fact that ability grouping was removed from the upper stage. Instead, schools transferred to internal differentiation within teaching groups, which was supported by the so-called time resource quota system. In mathematics teaching on the upper stage this meant smaller teaching groups (about 16-19 students) and it provided an opportunity for more individualized teaching and also more comprehensive use of applied problem solving than was possible earlier. On the other hand, the unfamiliarity and complexity of teaching mixed-ability groups constrained the spreading of applied problem solving as far and fast as had perhaps been thought.

Figure 5 sums up the approximate timing of the curriculum phases in question and the way they followed the North American reform trends. The figure differs from what was presented on page 26, in that it also tries to describe the development at the turn of the 1990s and to take a look at the future. The 'waves' of the figure attempt to describe the nature of the reform stages: first a strong rise and then a corresponding decline as a new wave emerges.

What is interesting in this development is, first of all, the speed at which the curriculum impacts are conveyed. The figure shows that reforms have always reached Finland after a certain time lag. In case of the New Math, the time lag was about ten years, in the Back to Basics-phase it was about five years, whereas a very quick reaction occurred to the Problem Solving-phase. Another important feature regarding these reform trends was that they were not transferred into the Finnish teaching practice as such, instead, they were transformed during the transfer process or shaped into a solution that fitted the domestic situation. Thus it was not only a question of borrowing an individual 'ideology' from some other country. In Finland, for example, the basic objectives and basic subject matter solutions were more thoroughly considered than in some extreme forms of the Back to Basics-thinking (e.g., Bracey 1983).

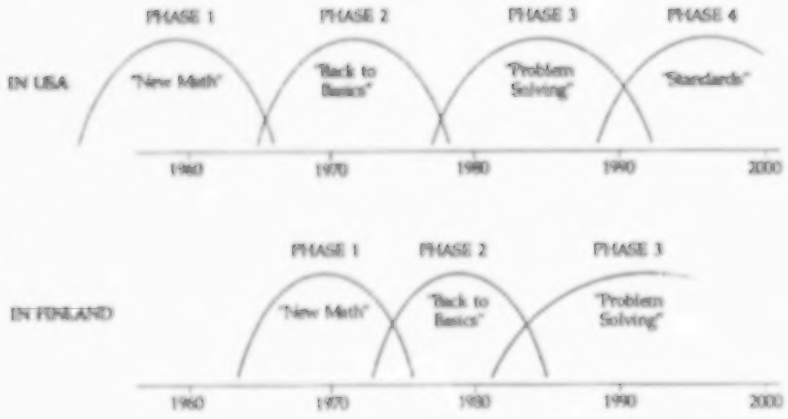

Figure 5. *The development phases of the Finnish comprehensive school mathematics curriculum related to the curricular trends in USA in 1994 (cf. Figure 3, p. 26).*

One explanation for the faster transfer of reform trends can be the general improvement of international communications and information transfer since the 1970s. The author believes, however, that more important causes are included in the general transfer mechanism by which these waves have reached us. For example, with the New Math the long time lag was probably caused by the thoroughness and radicalism of the reform. It created much discussion in all Western and Central European countries and also the Nordic countries worked in close cooperation and constructed their own joint solutions. All this took time. The fast spreading of the Problem Solving -wave, in turn, may be explained by the fact that it emerged as a counter-reaction to the Back to Basics -phase. A closer investigation of the transfer mechanism of the reforms would actually be an important topic for further research.

As regards this research in particular, it is a significant feature that in all phases of the curriculum development application and problem solving have been on the agenda: either explicitly in the teaching objectives or at least as issues to be considered in teaching. The position of applications in mathematics teaching has always been more clear and solid. As regards problem solving, the situation has been more difficult. Although the role of problem solving has been continuously strengthened in the curriculum, it has not yet extensively taken root in the practice of teaching. Very generally it consists of separate activities that are weakly integrated to teaching (e.g. Haapasalo 1985). Discussions with teachers indicate,

however, that more and more of them have internalized the importance of problem solving for mathematics teaching and have started using problem-centered working methods more frequently. This partly unorganized role of problem solving may also be the reason why the position of modelling in the curriculum and in the discussion on mathematics teaching has remained quite insignificant. Although modelling should be seen as an essential part of the applied problem solving process (see p. 5), it has been brought up in the curriculum only in a few isolated comments.

At present, it seems that in Finnish mathematics teaching the phase where applied problem solving has a central position continues. This comes clearly out in the 1994 principles of the curriculum plan for the comprehensive school published at the beginning of the year (Opetushallitus 1994). It is to be hoped that the local school-centered curricula succeed in concretising the goals and objectives presented in the principles in such a way that applied problem solving will be better manifested in practical teaching. The present phase in the USA mathematics education that began at the end of the 1980s could be called the phase of Mathematics Standards. The Curriculum and Evaluation Standards (National Council of Teachers of Mathematics 1989) strongly emphasize problem solving, communication, the importance of deduction and a strong self-concept in mathematics. Also in the Professional Standards for Teaching Mathematics (National Council of Teachers of Mathematics 1991) the contextual and problem-centered approach is dominating. Thus, this latest 'phase' can be seen as a clear continuation of the problem solving period of the 1980s.

5.2. What was the position of applications and in what way were applications used on the upper stage of the comprehensive school (especially at the 9th grade) at the beginning of the 1980s?

The results related to this research question are based on two empirical research data from the early 80s and on Burkhardt's conceptual frame of reference where applications were examined from the point of view of subject matter, teaching practice and learner. The results were discussed in Chapters 4.2. and 4.3.

At the turn of the 1980s mathematical application tasks were mainly understood as word problems, which were used as means of illustrating computation techniques. The echoes of the basic objectives and basic teaching contents still dominated the teaching of mathematics, and thus a problem-centered way of teaching that uses real application situations had very little foothold. Teaching was realized according to the *islands-approach*, so that students first studied 'pure' mathematics within a topic and then utilized it by means of applications and problem tasks.

Four out of five mathematics teachers considered the teaching of verbal tasks difficult, by which they mainly referred to the fact that these tasks were troublesome and time-consuming. The teachers were also unanimous about the importance

of teaching verbal tasks. They strongly emphasized that mathematics should be applied to everyday situations, whereas surprisingly little emphasis was given to application to other school subjects. Also more than half of the sixth and ninth grade teachers gave their students homework that required application of the topics treated in class.

Comprehensive school seventh graders thought that applications were both important and easy. On the basis of that fact it was considered that students on the upper stage are able and want to work with more demanding and real application tasks. When student achievements and their differences were examined in the tasks of different levels (i.e. computation, comprehension and application tasks), there were only small differences. The results seemed to show that this 'evenness' of the achievements was caused by the nature of the application tasks: they were either arithmetic operations clothed in a verbal form or typical cases of applications, which should be learned by heart through sufficient repetition.

Part of the teachers could even be described, on the oasis of their own reports, as teachers carrying out application-oriented teaching. However, the teaching followed the traditional method of teacher presentation and imitative practice, which contains serious problems in view of learning. Many studies actually show that even small deviations from the learned situations confuse the students. They are no longer able to perceive, analyze and model new types of situations, but their solution efforts are random and mechanical. Accordingly, it was not surprising that the teaching method in question was not found to correlate with student achievement so that this group's achievements would have been better than those of other teachers.

5.3. What efforts were made to increase applied problem solving in comprehensive school mathematics teaching during the 1980s?

The construction of the comprehensive school mathematics syllabus in 1980-81 gave a strong thrust to the increasing of problem solving and applications in teaching. It might even be said that the purpose of the authorities on mathematics education was to make the 80s a 'decade of problem solving'. On the other hand, because of the continuous changes, it was considered necessary to advance carefully, so that teachers' practical teaching work in schools would not be disturbed too much. It was believed that by adjusting the curriculum and learning materials and by organizing in-service training for teachers, the established goals could be gradually achieved. For this purpose the National Board of Education drafted a development plan for mathematics teaching.

The organization of special theme years was one of the measures of the development plan. In the 1983-84 school year application was the theme of mathematics teaching on the lower stage. The following school year was dedicated to the teaching of geometry, when special emphasis was given to the importance of geometry for a student's organization of his environment and for problem solving.

The educational policy decisions made in 1985 also had their effects. The reduction of mathematics lessons brought up the question in what way the subject matter would be trimmed and whether the decrease was focused again on the more time-consuming applied learning situations. The elimination of ability grouping from the upper stage of the comprehensive school meant that teachers had to face new kinds of teaching situations. As the size of teaching groups in mathematics became smaller and, for example, even two teachers could be used for teaching the same group, opportunities were created for examining real application situations in teaching. At the same time, however, teachers had to solve increasingly difficult problems of differentiation, because they had to teach mixed-ability teaching groups.

In mathematics teachers' in-service training problem solving was strongly visible throughout the whole latter part of the 80s. In 1986-87, for example, two national seminars were organized on "Problem solving in mathematics teaching". The seminar activity continued in 1989-90, when the theme was "Application in school mathematics" and the participants consisted mainly of the same people as in the earlier seminars. At the local level teachers' further training activities also increased: the number of separate in-service seminars organized by the Continuing Education Centers of universities grew rapidly, and they also started providing more long-term training.

Research and development work tried to do its own share for increasing applied problem solving, but the measures were rather haphazard. College supervisors and researchers worked as trainers in teachers in-service training and wrote a lot of the problems and future trends on mathematics education, about the necessity of problem solving in schools, about the factors connected to problem solving and of about the methodology in problem-centered teaching. No large-scale teaching experiments of applied problem solving were implemented, however.

At the end of the 80s the National Committee on Mathematics Teaching carried out a project for the improvement of the status of applications. This project first brought forth a collection of application tasks for the comprehensive school and upper secondary school, adopted from textbooks of different countries (Kouluhallitus 1988). Later on, a survey was published of academic Masters and doctoral theses related to applications in school mathematics (Kupari 1990). Also the Committee on Mathematics and Science Education working in 1987-89 took a strong stand in its final report in favour of problem solving, applications and modelling (Matemaattis-luonnontieteellisen perussivistyksen komitean loppumietintö 1989, 33):

> *"The focus of teaching should be moved from the practice of routine skills to the development of thinking. Efforts must be made to change the teaching climate in a more open and problem-centered direction. Students must learn to observe problems and meet problem situations in a way that induces them to find solutions and creates confidence in their own abilities...*
> *Students should be given a picture of mathematics as a general instrument in whose*

application modelling has a key position. It is especially in this connection that computers can be utilized."

The above-described training, research and development work were aimed to promote applied problem solving so that it would have a clearly stronger role in mathematics education than earlier. It was also attempted to increase teachers' knowledge of the importance of applied problem solving in teaching and thus to influence their attitudes. This was also believed to encourage teachers to develop and try out their own practical solutions. The efficiency of this activity was, however, weakened by the fact that the training occurred on a very small scale and the development projects were isolated and did not offer sufficiently concrete models for teaching (cf. the third international trend: increasing unification, pp. 15-16).

5.4. In what way did the studying and learning of applied problem solving change by the year 1990?

The Second National Assessment study implemented in 1990 and its body of research data on comprehensive school mathematics teaching revealed interesting information about applications and problem solving and about changes that have taken place in their studying and learning. The issue was examined more closely in Chapter 4.4.

In 1990 mathematics teachers held different conceptions about application than eleven years earlier. The teachers now set more rigorous criteria for application tasks than in 1979: an application task was now assessed also in regard to other characteristics than mere presentation in a verbal form. This indicated the fact that teachers' attitudes to applications had changed during the 80s. The result is very important, because the change that takes place in teacher's thinking is precisely what is considered to be the precondition for change of activities (cf. Janssens 1993). One might also assume that the development measures described in the previous paragraph had some impact on this change in the teachers' way of thinking.

In general, a majority of teachers thought that mathematics teaching had become more difficult and more strenuous since 1985. According to teachers, this could be seen in the fact that teaching had become more narrow and superficial, which was seen to be mostly caused by the too great heterogeneity of teaching groups. Furthermore, mathematics teachers thought that the emphasis on application and thinking skills in teaching had somewhat increased although simultaneously almost as much emphasis as in 1979 was put on the practice of computation skills. The increased share of application was seen especially in homework tasks: teachers now gave more regularly applications as homework to their students. This was possible, in turn, because the number of such tasks had clearly increased in the textbooks (cf. the second international trend: increasing globality, pp. 14-15). In their own opinion, the teachers emphasized problem

solving skills rather a lot in their teaching.

Student achievements improved on all three grade levels in the content area of applied mathematics, when identical tasks were compared. One explanation for this may simply be the increased number of application and problem solving tasks in learning materials. The task-specific examination of the content area on the ninth grade also revealed that in the mixed-ability teaching groups the learning opportunities of weaker students had become more versatile and these students had been able to learn new kinds of things. At the same time, there were signs that it had not been possible to offer the best students enough learning tasks corresponding to their capacities.

5.5. What factors have constrained applied problem solving from achieving the position it should have in the comprehensive school mathematics education?

In the development programs and written curriculum plans of mathematics education applied problem solving received an important position during the 1980s. But, concerning the situation in schools, there is a considerable gap between the far-reaching acceptance of applications, modelling and problem solving in the educational discussion and a low actual importance in school practice (Kaiser-Messmer 1989, Schupp 1989). Therefore, we have to ask which are the causes and obstacles for the small effects in practical mathematics teaching despite all the lofty goals and considerable efforts? There are numerous reasons for that and they are connected to a whole lot of issues: the organisation of school, the syllabi, teaching practices and materials, teachers and teacher training, and the learner. These issues were discussed to certain extent in Chapter 4.2.

- The organisation of the educational system in general and of the teaching practice in particular - the distribution of subject matter across lessons and the so-called "task didactics", where mathematics teaching is adjusted to solving many small problems - produces an unfavourable environment for applied problem solving (cf. Schupp 1989).

- The most important obstacle from the point of view of curriculum has probably been the fact that the general goals and grade-specific goals of mathematics teaching have been too isolated from each another. When important general objectives have been broken down into objectives for the different school years, they have changed into content lists of teaching or learning. What has been most problematic, the connections between the general goals and content goals have not been described when the original objective has lost a lot of its comprehensiveness, openness and process nature. When constructing the principles of the 1994 mathematics curriculum, much consideration was given to providing suggestions in the curriculum text concern-

ing versatile approaches and working methods (the so-called process objectives; Haapasalo & Kupari 1992).

- Many educators think that there is no structure in applied mathematics. Application and problem situations are not well-defined; they are messy, they have many answers, they involve estimation etc. Many teachers even doubt whether they belong to mathematics education at all. However, these characterisations do not hold universally because there are well-defined, elegant applications with single exact answers. Likewise, there is pure mathematics which is messy and has many answers.

  Building a curriculum in applied problem solving requires the structuration of subject matter. Mathematics teachers need a content structure into which they can fit application and problem tasks. This structure has to be more descriptive and richer than merely "Here is an application of linear equations" and there are different ways to do that (see e.g. Usiskin 1991).

- Lack of time has certainly been the most important obstacle from the instructional point of view, as real application and problem situations demand a lot of time. The shortage of time has been manifested at least in three ways. As the utmost consequence applied problem solving is not considered to belong to mathematics instruction at all, but extra-mathematical problems should be treated in the teaching of other subjects; for example elections in social sciences and shot putting in sports (cf. Blum 1991).

  A very general solution has also been the cutting down of subject matter. At the turn of the 80s, at least, the discrepancy between the teaching contents and teaching time led to elimination of tasks and applications related to concrete situations.

  The third implication has been the fact that it has not been possible to present applications and problem situations in the same way they arise in practical life. The tasks are often moulded into an easier form and thus become typical tasks with standard solutions. With very little effort the student can guess what it is that he should do with the figures of the task. This has partly been a consequence of the relatively small use of technical devices (calculators and computers) during the study process. In the lower grades of the comprehensive school calculators help, for instance, to make the measurement values of application tasks realistic and later on their use offers more extensive possibilities in instruction (cf. the fourth international trend: an extended involvement of computers, pp. 16-17).

- The textbooks, which teachers regard as their curriculum plans, have also supported mathematics teaching that is method-centered and takes place through small tasks. Almost all publishers have issued textbooks which have become perfect and teacher-proof packages that advance by following the textbook's page opening/lesson principle and contain a wide collection of

tasks and ready-made tests. Such textbooks certainly facilitate the smooth flow of teaching and, for example, the inclusion of application tasks in teaching. But it is very difficult - almost impossible - to include examination of actual problem situations, as this kind of work demands that teachers should create more open learning situations, utilize the possibilities of different advancement routes offered by students and use different assessment methods. Nevertheless, new types of learning materials and different additional materials have been increasingly produced during the past few years.

- For some teachers the insufficient use of applied problem solving can simply be due to the fact that they do not have enough adequate tasks at their disposal. A part of mathematics teachers received their education during time when it did not include courses in applied mathematics. That is why the offering of good applications and problems does not work as a solution for these teachers but it may even bring more frustration (cf. Usiskin 1991).

- The inclusion of applied problem solving in instruction will change mathematics teaching a lot and thus means new challenges for teachers. Teaching becomes more demanding because additional non-mathematical knowledge and qualifications are required. Many teachers also feel insecure in handling application situations that are not familiar to them. In contrast to mathematical tasks, real life problems are not complete. What is given and what is expected need not be clear, but can be a part of the problem, even its cause. Therefore, the arguments and answers are very subjective and provisional compared with solutions in computational mathematics. Teaching will also become more open and necessitates more variety of classroom interaction which is not very usual for mathematics teachers. For example, in the case of traffic flow, discussions about environment problems and their consequences would be necessary. All these might, however, undermine teacher's expert authority (Blum 1991). The openness of instruction causes that the assessment of student achievements and their knowing becomes especially difficult, because ordinary exams are not enough and other suitable evaluation tools are not available.

  The possibilities for helping the teachers from outside (with directions, booklets and books) are limited. The ways in which genuine real world problems can be posed and discussed in the classroom depends strongly on the particular conditions of each class.

- From the learner's point of view, mathematics studying will also become more demanding and less predictable but at the same time, probably, more meaningful and interesting. Realistic applications do indeed require more thinking, experience and maturity of the students than imitative tasks. Routine tasks - like mechanical computations - are of course more secure for many students, because they are easier to tackle and they can be solved merely by following certain 'recipes'. It is quite understandable that many students take

a cautious attitude to more demanding tasks, since they have not had an opportunity in any phase of their studies to get acquainted with them in meaningful situations.

Many teachers think that applications and real life problems are too difficult for most students whereas they will introduce them only for the best students. There are also some teachers who think that applications are not real mathematics and are appropriate for poorer students because the poorer students cannot learn proper mathematics. Mostly, the average students will receive fewer experiences with applications and real life problems.

5.6. Evaluations on the changes during the 1980s

Substantial changes occurred during the 1980s in Finnish comprehensive school mathematics and also in the teaching of applied problem solving. Many different ways were employed to promote the development of skills of using mathematics and to overcome obstacles, in order to increase applied problem solving in teaching.

On the basis of the obtained results and experiences, the development that has taken place in the learning and studying of applied problem solving has been positive in many ways. It is true that the changes have been cautious and they have taken place much more slowly than had been planned. That phase was restricted mostly to the pedagogical discussion and the activity of some schools and motivated teachers (cf. Schupp 1989). Many obstacles related to the organisation of instruction, the curriculum and the teaching culture constrained a faster and more far-reaching progress. We can by no means speak of any 'decade of problem solving'.

Applications and problem solving were often being considered both in discussions and in development endeavours. By contrast, modelling of application and problem solving situations was not regarded as a process needing to be taught or at least discussion was almost non-existing. At the start of the decade, applied problem solving was intended to be realized in the curriculum mainly by means of *the islands approach*. In other words, both 'pure mathematics' and applications connected with it were examined in each content area. In the late 1980s, we moved more clearly to using *the mixed model* where applications, modelling and problem solving were utilized as often as possible (e.g., investigations, project work).

The positive side of development was perhaps most evident in the change that occurred in teachers' thinking. This was evident, for example, in how the interpretation of the concept of applications expanded during the 80s. Although applied problem solving was not yet part of everyday teaching for most teachers, in-service training seminars and the active discussion that had been going on had probably had the effect that it would seem more and more possible for teachers to adopt applied problem solving as an important part of teaching. Also the teachers' opinions related to the changes in mathematics teaching at the middle of 1980s

(Kupari 1994) supported these results.

How does the future of mathematics education and applied problem solving look like? To what extent does the present interest towards applied problem solving represent a radical shift in emphasis and how far it is merely a continuation of previous trends used to be called applied mathematics? How far will such curricular approaches be sufficient in which applied problem solving is introduced in mathematics teaching mainly by increasing the number of tasks and by organizing problem solving sessions, when time allows it? In what ways could we promote more integrated ways of introducing applied problem solving as an organic part of mathematics curriculum and teaching? These questions will be discussed in Chapter 5.7. both more generally and on the basis of some foreign examples.

5.7. Some development prospects in school mathematics education

One major purpose of mathematics education in school is to provide the mathematics needed for intelligent citizenship and for personal decision-making. What is wrong, at the moment, concerning the population's intelligent citizenship and decision-making? Pollak (1989) has analyzed the question incisively:

> *"People don't seem to be able to handle uncertain situations - those which are inherently stochastic in nature... People have difficulty reasoning from data. In fact they are afraid of data, and will tolerate outrageously misleading graphics in popular and even technical presentations. They cannot plan intelligently - never mind optimise!... How many people can figure out alternative life insurance plans, financial possibilities, or interest rates? Another failing is the inability to think about systems. People always look for simple solutions... They expect everything to behave linearly, there are no unforeseen consequences, no instabilities, no feedback loops and no overloads. They don't think discretely or algorithmically. Above all, they do not know that it is possible to model situations in the real world in a structural, quantitative, analytic, that is mathematical way."*

The primary objective for the restructuration of mathematics curriculum is to empower students mathematically, to give students opportunities to use mathematics in a realistic way, and to do mathematics for themselves (Burrill 1993). Mathematics and sciences would be treated as possible resources, tool boxes and means in problem solving activities where artificial problems are replaced by real problem situations and where the process of problem posing and presentation arises from students' own interests and concerns. Motivating mathematics through applications and problems is an excellent way to approach towards the objective and also reach it. Real problems and questions motivate real mathematics, and students can see for themselves the connections between topics. Students learn to

model situations mathematically and use various techniques to analyze these models.

From the viewpoint of society Keitel (1993) has described the characteristics of the renewed mathematics education. According to her, in the search for problem environments and contexts some new criteria are needed. The social importance of contexts and problems cannot any longer be the tasks of discipline experts alone, since interdisciplinary and holistic approaches call for the common efforts of different expert groups (the users, the public, teachers and parents). Through redefining school subjects and 'subject empires' school subjects will broaden beyond the traditional boundaries. When modelling and project work are the major parts of teaching and learning, knowledge has to be justified by its pedagogical and social appropriateness, and the old hierarchies and closed systems of knowledge are replaced by flexible modules of contexts. As teachers change their traditional roles and use the above principles and guidelines for instruction, they can create mathematics activities which are interesting and exiting for students. Such forms of activity foster cooperation and support collective learning where the teacher is understood as interpreter and critic, instead of as a knowledge transmitter, who is bound into the process of knowledge generation and production. (Keitel 1993).

In the following the author introduces with the experiences from three countries what kind of efforts have been made in those countries to make applied problem solving a firmer part of school mathematics teaching. After that, a brief description will be more given about the direction along which the Finnish school mathematics is proceeding towards the turn the year 2000.

5.7.1. *An Australian example*

> *"What is it that we really value and how can it best be achieved within the constraints of ordinary school life?" (Stacey & Groves 1989).*

Stacey & Groves (1989) have analyzed Australian mathematics education and they think that there has been a considerable change from 1984 to 1988. Problem solving (including applications and modelling) has changed from a fashionable activity to a formally recognised part of the curriculum aims of most schools and states. Problem solving is seen as the global process with which most of the essential mathematical concepts and processes can be drawn on. At first problem solving appeared in primary and junior secondary curricula but afterwards also in the curriculum of the upper secondary. In different states and territories own curricular solutions have been developed in which the important role of problem solving can be seen both in having more time and in assessment. A lot of effort has been put on the active development of teaching approaches and it has also produced a bank of illustrative lesson ideas. For example, one set of lessons collects together investigations and problem solving and one set is on mathematical modelling.

There has also been substantial activity in many individual schools. One remarkable feature of the grass-roots approach has been the spread of Mathematics Task Centres (particularly for students in years 5 to 8). The tasks are concretely presented, they are stored ready for use without further teacher preparation and students work on the tasks in small groups.

The official changes for increasing problem solving (the so-called institutionalisation) have also given rise to serious challenges especially in the areas of assessment, definitions of goals and descriptions of student progress and content/process balance. If it is seen as important that students should plan their own investigations and follow their own paths, how can one student's work be compared with that of another student? Efforts have made to solve this, among other things, by outlining a set of criteria or guidelines for making any type of investigation, but many of the difficulties are also included in such schemes. How, for instance, can long pieces of work be assessed fairly if students receive input from a variety of sources: from teacher, parents and friends? It is also difficult to communicate to teachers the real aims of teaching for process and to ensure that all teachers recognise and value the desired objectives in their students' work.

In 1984 Stacey & Groves (1989) presented the following recommendations for advancing problem solving in schools: 1) the need for good teaching materials, 2) the need to investigate and promote appropriate teaching styles, 3) teacher education for problem solving, 4) definition of values and goals, 5) the need of syllabus statements to recognise process objectives and the conditions under which they may be achieved, 6) the need for research into questions arising from practice, and 7) the need for system-wide support. The researchers think that since 1984 considerable progress had taken place on most of these recommendations and at the end of 1980s the most urgent questions were those arising from assessment.

5.7.2. *An English example*

> *"The introduction of applications of mathematics, and of modelling in particular, into the mathematics curriculum presents most teachers with a formidable challenge" (Burkhardt 1989).*

With the help of the public examination system England has aimed at the promotion of well-supported gradual change in mathematics education. Burkhardt (1989) has described this development.

In the middle of the 1980s a new modular approach was developed to syllabus change. The key principle of this Testing Strategic Skills programme was a cautious realistic approach to introduce new examination tasks that would encourage curriculum changes that are widely recognised as desirable. The main elements in implementing this approach were the following:

* *Gradual change.* The idea was to change the syllabus to the extent of three weeks' teaching in any year (about 5% of the syllabus). The modular approach was used because it is difficult for teachers to substantially change their own work and especially when rather different methods of working are involved. The so-called sweeping syllabus reforms have often had limited success, either because they have been taken up only by a small minority, or because they have been confined to details of subject content (the easiest changes to absorb).

* *Effective support.* If it is a purpose that teachers begin to examine less familiar topic areas or modes of classroom learning activity, it is important to offer them something equally supportive. The module teaching package was developed to do this. The package contains a range of approaches and materials for teachers to choose from, or to develop themselves (teachers will do things in their own way in their own classroom).

* *Empirical classroom development.* In general, it is not possible to guess reliably what the effects of any significant change will be and the outcomes are often quite different from the original intentions. In this development programme these dangers were much reduced by an empirical approach in which all the elements of the module were developed and tested with the broad representative samples of teachers and children.

5.7.3. *A German example*

"Students should regard themselves as 'wanderers between two worlds' - reality and mathematics - and appreciate the importance of knowledge and understanding in both worlds in order to cope with real-world problems" (Schupp 1989).

Schupp (1989) has described German experiences how to change the practice of mathematics teaching in the direction which utilises applied problem solving.
He thinks that the mathematics educators and didacticians have to come more in contact with the teachers in the schools. Working together they should plan, carry out and evaluate appropriate short- and long-term projects. The question of development and change is directed above all at the mathematics teachers themselves, too. Schupp (1989) mentions, among other things, following aspects which will promote applied mathematics teaching and can be used easily:

- Those parts of the curriculum should be extended where application and problem situations are constitutive at the expense of other sections which merely provide techniques for situations to come. These are, for instance, stochastics and some parts of geometry and of algebra.

- It must be clearly stated that many central mathematical concepts of the lower

secondary school are also good as standard models. This holds, in particular, for the concept of function.

- Students' everyday life and their fields of activity as well as other school subjects must be taken into consideration in an appropriate way. This does not exclude the preference of areas in which the teacher has specific competencies and/or in which the students are especially interested.

- It is necessary to change consciously the viewpoint during the process of application. We cannot think only in the ordinary way that at a time a piece of mathematics is applied to a well-prepared piece of reality. Students must also learn to notice that there may be several fitting models for the same situation, that they now have to combine parts of mathematics rather isolated till now and that they first have to construct the mathematical means which are helpful in the given situation, and so on.

- There would seem to be no insurmountable difficulties in introducing the applied problem solving process (cf. Figure 1, p. 5) rather early. The steps of the process may be introduced as reactions to heuristic questions such as 'Did you really answer the question?', 'Is this a fitting answer?', 'Can we improve the solution?', 'What don't we know yet?'.

- A deeper understanding of the relations between situation and model yet requires some maturity and the ability to think in relative and evaluating terms, which the students acquire only by the end of lower secondary school. However, the teacher should not plan special lessons for the examination of these relations in advance, but be prepared always when the corresponding questions emerge by themselves while the students discuss a knotty problem.

5.7.4. *The Finnish approach*

In Finland, a new kind of curricular approach was outlined during 1990-91, when the committee, *Working Group for the Development of Mathematics Education*, appointed by the National Board of Education was working. In its report the group presented outlines for mathematics teaching in the form of development principles (Opetushallitus 1991). In the following paragraphs the two first principles are presented in order to describe concisely the trend based on the constructivist learning conception and the role of applied problem solving in it. The work and results of the group have been discussed in more detail by Haapasalo (1990), Haapasalo & Kupari (1991, 1992) and Kupari (1993b, 1994).

Principle 1. *Mathematics should be understood more broadly than only as the learning of certain computation skills. Acquisition of mathematics should be linked with the*

student's mental growth, and with increasing the extent and logic of his thinking, as well as with the formation of his world view. Problem solving should be a central teaching principle, in addition to the mathematical and logical requirements.

Principle 2. *The primary goal of mathematics teaching should consist of the development of the student's ability to classify, organize and model situations that confront him in the surrounding world. Consequently, the starting point of teaching should not consist of prestructured information, which is presented as given to the students. Instead, by emphasizing heuristic information acquisition process, the aim should be that the student's knowledge and skills are gradually organized and focused into a useful structure.*

In January 1994, the National Board of Education published the new Principles of the Curriculum in the Comprehensive School (Opetushallitus 1994). At the same time, they signify a start for a new type of educational and curricular culture in schools. The Principles of the Curriculum are the foundation on which the municipalities and the schools locally prepare their own curriculum, which describes and develops instruction and guides the teaching activities in practice. In certain respects the local curricula interpret, modify and complement the national Principles. Therefore the objectives and contents of different subjects have been presented briefly and rather generally in the Principles. The process for making curricula in schools and municipalities is in progress and a certain part of schools are prepared to implement their curricula since the autumn 1994.

In the section of the Principles concerning mathematics education the importance of applications, modelling and problem solving is expressed as an objective which follows quite closely the text presented by the above-mentioned Working Group (principle 2):

"The goal of mathematics teaching in the comprehensive school is, above all, to develop the student's ability to classify, organize, and model situations that confront him in the surrounding world with the concepts acquired earlier" (Opetushallitus 1994).

Furthermore, problem solving is understood as a central teaching principle. However, it is the results of curriculum work completed by the schools and municipalities themselves that indicate in what way mathematics education will develop and what role and form applied problem solving will have in teaching. The next few years will show that.

It is very encouraging, in view of the year 2000, that both researchers and school teachers have shown active interest in the improvement of the status of applied problem solving in teaching. Many researchers have dealt with these themes from the perspective of the curriculum theory, teaching methods, learning materials and assessment (e.g., Björkqvist 1993, Enkenberg 1990, Haapasalo 1991,

Leino 1991, Keranto 1992, Kupari 1993, Pehkonen 1991). The time is indeed most appropriate for discussions and introduction of new ideas.

5.8. Final remarks

There seems to be a clear unanimous agreement about the fact that applied problem solving should be an essential part of mathematics education at all levels of schooling. Examples and materials are available to a sufficient extent. Also, the empirical investigations have produced encouraging results with regard to various positive effects of applied problem solving for students and learning. Furthermore, the results have shown that through the inclusion and increase of applied problem solving in teaching we can better comprehend the world around us, more deeply and more extensively understand mathematical concepts, advance the development of formal abilities and improve attitudes towards mathematics by giving more meaning to study it. Strengthening the position of applied problem solving makes, of course, mathematics teaching more demanding and more challenging both for teachers and students but it may not be used as an argument not to change mathematics teaching in this direction. Emphasis on the relation between real world and mathematics in not any fortuitous fashion but a permanent and indisputable challenge to which more and more teachers, fortunately, admit to engage.

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The problem-centered approach is becoming a more and more important principle in teaching school mathematics in Finland. The study discusses the role and development of applied problem solving in mathematics education of the comprehensive school in the 1980s. The theme is described in a wider international context, and as a result of this analysis some thoughts for the development of mathematics education will also be presented.

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